Foundations for Dynamic Equipment

Reported by ACI Committee 351

James P. Lee* Chair

Yelena S. Golod† Secretary

William L. Bounds†

William D. Brant

Shu-jin Fang

Shraddhakar Harsh

Charles S. Hughes

Erick Larson

Fred G. Louis

Jack Moll

Ira W. Pearce

Andrew Rossi

Robert L. Rowan, Jr.

William E. Rushing, Jr.

Abdul Hai Sheikh

Anthony J. Smalley

Philip A. Smith

W. Tod Sutton†

F. Alan Wiley

This report presents to industry practitioners the various design criteria and methods of analysis, design, and construction applied to dynamic equipment foundations.

Keywords: amplitude; concrete; foundation; reinforcement; vibration.

CONTENTS

Chapter 1—Introduction, p. 351.3R-2

1.1—Background

1.2—Purpose

1.3—Scope

1.4—Notation

Chapter 2—Foundation and machine types, p. 351.3R-4

2.1—General considerations

Chapter 3—Design criteria, p. 351.3R-7

2.2—Machine types

2.3—Foundation types

Chapter 4—Design methods and materials, p. 351.3R-26

3.1—Overview of design criteria

3.2—Foundation and equipment loads

3.3—Dynamic soil properties

3.4—Vibration performance criteria

3.5—Concrete performance criteria

3.6—Performance criteria for machine-mounting systems

3.7—Method for estimating inertia forces from multi-cylinder machines

Chapter 5—Construction considerations, p. 351.3R-53

4.1—Overview of design methods

4.2—Impedance provided by the supporting media

4.3—Vibration analysis

4.4—Structural foundation design and materials

4.5—Use of isolation systems

4.6—Repairing and upgrading foundations

4.7—Sample impedance calculations

It is the responsibility of the user of this document to establish health and safety practices appropriate to the specific circumstances involved with its use. ACI does not make any representations with regard to health and safety issues and the use of this document. The user must determine the applicability of all regulatory limitations before applying the document and must comply with all applicable laws and regulations, including but not limited to, United States Occupational Safety and Health Administration (OSHA) health and safety standards.
Chapter 6—References, p. 351.3R-57

6.1—Referenced standards and reports
6.2—Cited references
6.3—Software sources and other references
6.4—Terminology

CHAPTER 1—INTRODUCTION

1.1—Background

Heavy machinery with reciprocating, impacting, or rotating masses requires a support system that can resist dynamic forces and the resulting vibrations. When excessive, such vibrations may be detrimental to the machinery, its support system, and any operating personnel subjected to them.

Many engineers with varying backgrounds are engaged in the analysis, design, construction, maintenance, and repair of machine foundations. Therefore, it is important that the owner/operator, geotechnical engineer, structural engineer, and equipment supplier collaborate during the design process. Each of these participants has inputs and concerns that are important and should be effectively communicated with each other, especially considering that machine foundation design procedures and criteria are not covered in building codes and national standards. Some firms and individuals have developed their own standards and specifications as a result of research and development activities, field studies, or many years of successful engineering or construction practices. Unfortunately, most of these standards are not available to many practitioners. As an engineering aid to those persons engaged in the design of foundations for machinery, the committee developed this document, which presents many current practices for dynamic equipment foundation engineering and construction.

1.2—Purpose

The committee presents various design criteria and methods and procedures of analysis, design, and construction currently applied to dynamic equipment foundations by industry practitioners.

This document provides general guidance with reference materials, rather than specifying requirements for adequate design. Where the document mentions multiple design methods and criteria in use, factors, which may influence the choice, are presented.

1.3—Scope

This document is limited in scope to the engineering, construction, repair, and upgrade of dynamic equipment foundations. For the purposes of this document, dynamic equipment includes the following:

1. Rotating machinery;
2. Reciprocating machinery; and
3. Impact or impulsive machinery.

1.4—Notation

\[
\begin{align*}
K & = \text{stiffness matrix} \\
K' & = \text{impedance with respect to CG} \\
k & = \text{reduced stiffness matrix} \\
k' & = \text{battered pile stiffness matrix} \\
M & = \text{mass matrix} \\
m & = \text{reduced mass matrix} \\
T & = \text{transformation matrix for battered pile} \\
\alpha_{ij} & = \text{matrix of interaction factors between any two piles with diagonal terms } \alpha_{ii} = 1 \\
A & = \text{displacement amplitude} \\
A_{\text{head}}, A_{\text{crank}} & = \text{head and crank areas, in}^2 \text{ (mm}^2) \\
\alpha & = \text{plan dimensions of a rectangular foundation} \\
\alpha_i & = \text{dimensionless frequency} \\
B_i & = \text{cylinder bore diameter, in. (mm)} \\
B_i' & = \text{mass ratio for the } i\text{-th direction} \\
B_p & = \text{ram weight, tons (kN)} \\
b_1, b_2 & = 0.425 \text{ and } 0.687, \text{ Eq. (4.15d)} \\
c_{gi} & = \text{damping of pile group in the } i\text{-th direction} \\
c_i & = \text{damping constant for the } i\text{-th direction} \\
c_i (adj) & = \text{damping in the } i\text{-th direction adjusted for material damping} \\
c_{ij} & = \text{equivalent viscous damping of pile } j \text{ in the } i\text{-th direction} \\
D_i & = \text{damping ratio for the } i\text{-th direction} \\
D_{\text{rod}} & = \text{rod diameter, in. (mm)} \\
d & = \text{pile diameter} \\
d_p & = \text{nominal bolt diameter, in. (m)} \\
d_s & = \text{displacement of the slide, in. (mm)} \\
E_p & = \text{Young’s modulus of the pile} \\
e_m & = \text{mass eccentricity, in. (mm)} \\
e_v & = \text{void ratio} \\
F & = \text{time varying force vector} \\
F_1 & = \text{correction factor} \\
F_{\text{block}} & = \text{the force acting outwards on the block from which concrete stresses should be calculated, lbf (N)} \\
(F_\text{bolt})_{\text{CHG}} & = \text{the force to be restrained by friction at the cross head guide tie-down bolts, lbf (N)} \\
(F_\text{bolt})_{\text{frame}} & = \text{the force to be restrained by friction at the frame tie-down bolts, lbf (N)} \\
F_D & = \text{damper force} \\
F_{\text{GMAX}} & = \text{maximum horizontal gas force on a throw or cylinder, lbf (N)} \\
F_{\text{IMAX}} & = \text{maximum horizontal inertia force on a throw or cylinder, lbf (N)} \\
F_o & = \text{dynamic force amplitude (zero-to-peak), lbf (N)} \\
F_r & = \text{maximum horizontal dynamic force} \\
F_{\text{red}} & = \text{a force reduction factor with suggested value of 2, to account for the fraction of individual cylinder load carried by the compressor frame ("frame rigidity factor")} \\
F_{\text{rod}} & = \text{force acting on piston rod, lbf (N)} \\
F_{\text{rod}} & = \text{dynamic inertia force of slide, lbf (N)} \\
F_{\text{THROW}} & = \text{horizontal force to be resisted by each throw’s anchor bolts, lbf (N)} \\
F_{\text{unbalance}} & = \text{the maximum value from Eq. (3.18)} \\
\end{align*}
\]
$f_{11}, f_{22}$ = dimensionless stiffness and damping functions for the $i$-th direction, piles

$f_m$ = frequency of motion, Hz

$f_n$ = system natural frequency (cycles per second)

$f_o$ = operating speed, rpm

$G$ = dynamic shear modulus of the soil

$G_{ave}$ = the average value of shear modulus of the soil over the pile length

$G_c$ = the average value of shear modulus of the soil over the critical length

$GE$ = pile group efficiency

$G_t$ = soil shear modulus at tip of pile

$G_J$ = torsional stiffness of the pile

$G_s$ = dynamic shear modulus of the embedment (side) material

$G_z$ = the shear modulus at depth $z = l_c/4$

$H$ = depth of soil layer

$I_i$ = mass moment of inertia of the machine-foundation system for the $i$-th direction

$I_p$ = moment of inertia of the pile cross section

$i$ = a directional indicator or modal indicator, Eq. (4.48), as a subscript

$K_{2}$ = a parameter that depends on void ratio and strain amplitude

$K_i^{eff}$ = the effective bearing stiffness, lbf/in. (N/mm)

$K_{ij}$ = impedance in the $i$-th direction with respect to motion of the CG in $j$-th direction

$K_n$ = nut factor for bolt torque

$K_{mu}$ = horizontal spring constant

$K_{wu}$ = coupling spring constant

$K_{wq}$ = rocking spring constant

$k$ = the dynamic stiffness provided by the supporting media

$k_{ei}$ = impedance in the $i$-th direction due to embedment

$k_{ij}$ = pile group stiffness in the $i$-th direction

$k_j$ = stiffness for the $i$-th direction

$k_{(adj)}$ = stiffness in the $i$-th direction adjusted for material damping

$k_i^*$ = complex impedance for the $i$-th direction

$k_i^{(adj)}$ = impedance adjusted for material damping

$k_{ij}$ = stiffness of pile $j$ in the $i$-th direction

$k_{ij}$ = battered pile stiffness matrix

$k_r$ = stiffness of individual pile considered in isolation

$k_{st}$ = static stiffness constant

$k_{ij}$ = vertical stiffness of a single pile

$L$ = length of connecting rod, in. (mm)

$L_B$ = the greater plan dimension of the foundation block, ft (m)

$L_i$ = length of the connecting rod of the crank mechanism at the $i$-th cylinder

$l$ = depth of embedment (effective)

$l_c$ = critical length of a pile

$l_p$ = pile length

$M_n$ = hammer mass including any auxiliary foundation, lbm (kg)

$M_r$ = ram mass including dies and ancillary parts, lbm (kg)

$m$ = mass of the machine-foundation system

$m_d$ = slide mass including the effects of any balance mechanism, lbm (kg)

$m_r$ = rotating mass, lbm (kg)

$m_{rec,i}$ = reciprocating mass for the $i$-th cylinder

$m_{rot,i}$ = rotating mass of the $i$-th cylinder

$m_s$ = effective mass of a spring

$(N_{bol})_{CHG}$ = the number of bolts holding down one crosshead guide

$(N_{bol})_{frame}$ = the number of bolts holding down the frame, per cylinder

$NT$ = normal torque, ft-lbf (m-N)

$P_{head}, P_{crank}$ = instantaneous head and crank pressures, psi (µPa)

$P_s$ = power being transmitted by the shaft at the connection, horsepower (kilowatts)

$R, R_i$ = equivalent foundation radius

$r_i$ = length of crank, in. (mm)

$r_l$ = radius of the crank mechanism of the $i$-th cylinder

$r_o$ = pile radius or equivalent radius

$S$ = press stroke, in. (mm)

$S_f$ = service factor, used to account for increasing unbalance during the service life of the machine, generally greater than or equal to 2

$S_{11}, S_{22}$ = dimensionless parameters (Table 4.2)

$s$ = distance between piles

$T$ = foundation thickness, ft (m)

$T_b$ = bolt torque, lbf-in. (N-m)

$T_{min}$ = minimum required anchor bolt tension

$t$ = time, s

$V_{max}$ = the maximum allowable vibration, in. (mm)

$V_i$ = shear wave velocity of the soil, ft/s (m/s)

$W$ = displacement amplitude

$V_i'$ = velocity, in./s (cm/s)

$V_h$ = post-impact hammer velocity, in./s (mm/s)

$V_o$ = reference velocity = 18.4 ft/s (5.6 m/s) from a free fall of 5.25 ft (1.6 m)

$W_r$ = ram impact velocity, ft/s (m/s)

$W$ = strain energy

$W_e$ = equipment weight at anchorage location

$W_f$ = weight of the foundation, tons (kN)

$W_p$ = bolt preload, lbf (N)

$W_r$ = rotating weight, lbf (N)

$w$ = soil weight density

$X$ = vector representation of time-dependent displacements for MDOF systems

$X_i$ = distance along the crankshaft from the reference origin to the $i$-th cylinder

$y$ = the pile coordinates indicated in Fig. 4.9

$x, z$ = pile location reference distances

$x_r, z_r$ = distance from the CG to the base support

$y_c$ = distance from the CG to the level of embedment resistance

$y_p$ = crank pin displacement in local Y-axis, in. (mm)
\[ Z_p = \text{piston displacement, in. (mm)} \]
\[ \varepsilon_p = \text{crank pin displacement in local Z-axis, in. (mm)} \]
\[ \alpha = \text{the angle between a battered pile and vertical} \]
\[ \alpha' = \text{modified pile group interaction factor} \]
\[ \alpha_t = \text{coefficient dependent on Poisson’s ratio as given in Table 4.1} \]
\[ \alpha_h = \text{ram rebound velocity relative to impact velocity} \]
\[ \alpha_i = \text{the phase angle for the crank radius of the } i\text{-th cylinder, rad} \]
\[ \alpha_{ij} = \text{complex pile group interaction factor for the } i\text{-th pile to the } j\text{-th pile} \]
\[ \alpha_{uf} = \text{the horizontal interaction factor for fixed-headed piles (no head rotation)} \]
\[ \alpha_{uH} = \text{the horizontal interaction factor due to horizontal force (rotation allowed)} \]
\[ \alpha_v = \text{vertical interaction coefficient between two piles} \]
\[ \alpha_{yH} = \text{the rotation due to horizontal force} \]
\[ \alpha_{yM} = \text{the rotation due to moment} \]
\[ \beta = \text{system damping ratio} \]
\[ \beta_j = \text{rectangular footing coefficients (Richart, Hall, and Woods 1970), } i = v, u, \text{ or } \psi \]
\[ \beta_j = \text{coefficient dependent on Poisson’s ratio as given in Table 4.1, } j = 1 \text{ to } 4 \]
\[ \beta_m = \text{material damping ratio of the soil} \]
\[ \beta_p = \text{angle between the direction of the loading and the line connecting the pile centers} \]
\[ \delta = \text{loss angle} \]
\[ \Delta W = \text{area enclosed by the hysteretic loop} \]
\[ e_{ir} = \text{the elements of the inverted matrix } [\alpha_{ij}]^{-1} \]
\[ \psi_i = \text{reduced mode shape vector for the } i\text{-th mode} \]
\[ \gamma_j = \text{coefficient dependent on Poisson’s ratio as given in Table 4.1, } j = 1 \text{ to } 4 \]
\[ \lambda = \text{pile-soil stiffness ratio } (E_p/G_l) \]
\[ \mu = \text{coefficient of friction} \]
\[ \nu = \text{Poisson’s ratio of the soil} \]
\[ \nu_s = \text{Poisson’s ratio of the embedment (side) material} \]
\[ \rho = \text{soil mass density (soil weight density/gravitational acceleration)} \]
\[ \rho_a = G_{ave}/G_l \]
\[ \rho_c = G_c/G_c \]
\[ \sigma_a = \text{probable confining pressure, lbf/ft}^2 \text{ (Pa)} \]
\[ \omega_i = \text{circular natural frequency for the } i\text{-th mode} \]
\[ \omega_m = \text{circular frequency of motion} \]
\[ \omega_n = \text{circular natural frequencies of the system} \]
\[ \omega_o = \text{circular operating frequency of the machine (rad/s)} \]
\[ \omega_{su}, \omega_{sv} = \text{circular natural frequencies of a soil layer in } u \text{ and } v \text{ directions} \]

CHAPTER 2—FOUNDATION AND MACHINE TYPES

2.1—General considerations

The type, configuration, and installation of a foundation or support structure for dynamic machinery may depend on the following factors:

1. Site conditions such as soil characteristics, topography, seismicity, climate, and other effects;
2. Machine base configuration such as frame size, cylinder supports, pulsation bottles, drive mechanisms, and exhaust ducts;
3. Process requirements such as elevation requirements with respect to connected process equipment and hold-down requirements for piping;
4. Anticipated loads such as the equipment static weight, and loads developed during erection, startup, operation, shutdown, and maintenance;
5. Erection requirements such as limitations or constraints imposed by construction equipment, procedures, techniques, or the sequence of erection;
6. Operational requirements such as accessibility, settlement limitations, temperature effects, and drainage;
7. Maintenance requirements such as temporary access, laydown space, in-plant crane capabilities, and machine removal considerations;
8. Regulatory factors or building code provisions such as tied pile caps in seismic zones;
9. Economic factors such as capital cost, useful or anticipated life, and replacement or repair cost;
10. Environmental requirements such as secondary containment or special concrete coating requirements; and
11. Recognition that certain machines, particularly large reciprocating compressors, rely on the foundation to add strength and stiffness that is not inherent in the structure of the machine.

2.2—Machine types

2.2.1 Rotating machinery—This category includes gas turbines, steam turbines, and other expanders; turbo-pumps and compressors; fans; motors; and centrifuges. These machines are characterized by the rotating motion of impellers or rotors.

Unbalanced forces in rotating machines are created when the mass centroid of the rotating part does not coincide with the center of rotation (Fig. 2.1). This dynamic force is a function of the shaft mass, speed of rotation, and the magnitude of the offset. The offset should be minor under manufactured conditions when the machine is well balanced, clean, and without wear or erosion. Changes in alignment, operation near resonance, blade loss, and other malfunctions or undesirable conditions can greatly increase the force applied to its bearings by the rotor. Because rotating machines normally trip and shut down at some vibration limit, a realistic continuous dynamic load on the foundation is that resulting from vibration just below the trip level.

2.2.2 Reciprocating machinery—For reciprocating machinery, such as compressors and diesel engines, a piston moving in a cylinder interacts with a fluid through the
kinematics of a slider crank mechanism driven by, or driving, a rotating crankshaft.

Individual inertia forces from each cylinder and each throw are inherently unbalanced with dominant frequencies at one and two times the rotational frequency (Fig. 2.2).

Reciprocating machines with more than one piston require a particular crank arrangement to minimize unbalanced forces and moments. A mechanical design that satisfies operating requirements should govern. This leads to piston/cylinder assemblies and crank arrangements that do not completely counter-rotate; therefore, unbalanced loads occur, which should be resisted by the foundation.

Individual cylinder fluid forces act outward on the cylinder head and inward on the crankshaft (Fig. 2.2). For a rigid cylinder and frame these forces internally balance, but deformations of large machines can cause a significant portion of the fluid load to be transmitted to the mounts and into the foundation. Particularly on large reciprocating compressors with horizontal cylinders, it is inappropriate and unconservative to assume the compressor frame and cylinder are sufficiently stiff to internally balance all forces. Such an assumption has led to many inadequate mounts for reciprocating machines.

2.2.3 Impulsive machinery—Equipment, such as forging hammers and some metal-forming presses, operate with regulated impacts or shocks between different parts of the equipment. This shock loading is often transmitted to the foundation system of the equipment and is a factor in the design of the foundation.

Closed die forging hammers typically operate by dropping a weight (ram) onto hot metal, forcing it into a predefined shape. While the intent is to use this impact energy to form and shape the material, there is significant energy transmission, particularly late in the forming process. During these final blows, the material being forged is cooling and less shaping takes place. Thus, pre-impact kinetic energy of the ram converts to post-impact kinetic energy of the entire forging hammer. As the entire hammer moves downward, it becomes a simple dynamic mass oscillating on its supporting medium. This system should be well damped so that the oscillations decay sufficiently before the next blow. Timing of the blows commonly range from 40 to 100 blows per min. The ram weights vary from a few hundred pounds to 35,000 lb (156 kN). Impact velocities in the range of 25 ft/s (7.6 m/s) are common. Open die hammers operate in a similar fashion but are often of two-piece construction with a separate hammer frame and anvil.

Forging presses perform a similar manufacturing function as forging hammers but are commonly mechanically or hydraulically driven. These presses form the material at low velocities but with greater forces. The mechanical drive system generates horizontal dynamic forces that the engineer should consider in the design of the support system. Rocking stability of this construction is important. Figure 2.3 shows a typical horizontal forcing function through one full stroke of a forging press.

Mechanical metal forming presses operate by squeezing and shearing metal between two dies. Because this equipment can vary greatly in size, weight, speed, and operation, the engineer should consider the appropriate type. Speeds can vary from 30 to 1800 strokes per min. Dynamic forces from the press develop from two sources: the mechanical balance of the moving parts in the equipment and the response of the press frame as the material is sheared (snap-through forces). Imbalances in the mechanics of the equipment can occur both horizontally and vertically. Generally high-speed equipment is well balanced. Low-speed equipment is often not balanced because the inertia forces at low speeds are small. The dynamic forces generated by all of these presses can be significant as they are transmitted into the foundation and propagated from there.

2.2.4 Other machine types—Other machinery generating dynamic loads include rock crushers and metal shredders. While part of the dynamic load from these types of equipment tend to be based on rotating imbalances, there is also a
random character to the dynamic signal that varies with the particular operation.

2.3—Foundation types

2.3.1 Block-type foundation (Fig. 2.4)—Dynamic machines are preferably located close to grade to minimize the elevation difference between the machine dynamic forces and the center of gravity of the machine-foundation system. The ability to use such a foundation primarily depends on the quality of near surface soils. Block foundations are nearly always designed as rigid structures. The dynamic response of a rigid block foundation depends only on the dynamic load, foundation’s mass, dimensions, and soil characteristics.

2.3.2 Combined block-type foundation (Fig. 2.5)—Combined blocks are used to support closely spaced machines. Combined blocks are more difficult to design because of the combination of forces from two or more machines and because of a possible lack of stiffness of a larger foundation mat.

2.3.3 Tabletop-type foundation (Fig. 2.6)—Elevated support is common for large turbine-driven equipment such as electric generators. Elevation allows for ducts, piping, and ancillary items to be located below the equipment. Tabletop structures are considered to be flexible, hence their response to dynamic loads can be quite complex and depend both on the motion of its discreet elements (columns, beams, and footing) and the soil upon which it is supported.

2.3.4 Tabletop with isolators (Fig. 2.7)—Isolators (springs and dampers) located at the top of supporting columns are sometimes used to minimize the response to dynamic loading. The effectiveness of isolators depends on the machine speed and the natural frequency of the foundation. Details of this type of support are provided in Section 4.5.

2.3.5 Spring-mounted equipment (Fig. 2.8)—Occasionally pumps are mounted on springs to minimize thermal forces from connecting piping. The springs are then supported on a block-type foundation. This arrangement has a dynamic effect similar to that for tabletops with vibration isolators. Other types of equipment are spring mounted to limit the transmission of dynamic forces.

2.3.6 Inertia block in structure (Fig. 2.9)—Dynamic equipment on a structure may be relatively small in comparison to the overall size of the structure. In this situation, dynamic machines are usually designed with a supporting inertia block to alter natural frequencies away from machine operating speeds and resist amplitudes by increasing the resisting inertia force.

2.3.7 Pile foundations (Fig. 2.10)—Any of the previously mentioned foundation types may be supported directly on soil or on piles. Piles are generally used where soft ground condi-
CHAPTER 3—DESIGN CRITERIA

3.1—Overview of design criteria

The main issues in the design of concrete foundations that support machinery are defining the anticipated loads, establishing the performance criteria, and providing for these through proper proportioning and detailing of structural members. Yet, behind this straightforward definition lies the need for careful attention to the interfaces between machine, mounting system, and concrete foundation.

The loads on machine foundations may be both static and dynamic. Static loads are principally a function of the weights of the machine and all its auxiliary equipment. Dynamic loads, which occur during the operation of the machine, result from forces generated by unbalance, inertia of moving parts, or both, and by the flow of fluid and gases for some machines. The magnitude of these dynamic loads primarily depends upon the machine’s operating speed and the type, size, weight, and arrangement (position) of moving parts within the casing.

The basic goal in the design of a machine foundation is to limit its motion to amplitudes that neither endanger the satisfactory operation of the machine nor disturb people working in the immediate vicinity (Gazetas 1983). Allowable amplitudes depend on the speed, location, and criticality or function of the machine. Other limiting dynamic criteria affecting the design may include avoiding resonance and excessive transmissibility to the supporting soil or structure. Thus, a key ingredient to a successful design is the careful engineering analysis of the soil-foundation response to dynamic loads from the machine operation.

The foundation’s response to dynamic loads can be significantly influenced by the soil on which it is constructed. Consequently, critical soil parameters, such as the dynamic soil shear modulus, are preferably determined from a field investigation and laboratory tests rather than relying on generalized correlations based on broad soil classifications. Due to the inherent variability of soil, the dynamic response of machine foundations is often evaluated using a range of values for the critical soil properties.

Furthermore, a machinery support structure or foundation is designed with adequate structural strength to resist the worst possible combination of loads occurring over its service life. This often includes limiting soil-bearing pressures to well within allowable limits to ensure a more predictable dynamic response and prevent excessive settlements and soil failures. Additionally, concrete members are designed and detailed to prevent cracking due to fatigue and stress reversals caused by dynamic loads, and the machine’s mounting system is designed and detailed to transmit loads from the machine into the foundation, according to the criteria in Section 3.6.

3.2—Foundation and equipment loads

Foundations supporting reciprocating or rotating compressors, turbines, generators and motors, presses, and other machinery should withstand all the forces that may be imposed on them during their service life. Machine foundations are unique because they may be subjected to significant dynamic loads during operation in addition to normal design loads of gravity, wind, and earthquake. The magnitude and characteristics of the operating loads depend on the type, size, speed, and layout of the machine.

Generally, the weight of the machine, center of gravity, surface areas, and operating speeds are readily available from the manufacturer of the machine. Establishing appropriate values for dynamic loads is best accomplished through careful communication and clear understanding between the machine manufacturer and foundation design engineer as to the purpose, and planned use for the requested information, and the definition of the information provided. It is in the best interests of all parties (machine manufacturer, foundation design engineer, installer, and operator) to ensure effective definition and communication of data and its appropriate use. Machines always experience some level of unbalance, vibration, and force transmitted through the bearings. Under some off-design conditions, such as wear, the forces may increase significantly. The machine manufacturer and foundation design engineer should work together so that their combined knowledge achieves an integrated system structure which robustly serves the needs of its owner and operator and withstands all expected loads.

Sections 3.2.1 to 3.2.6 provide commonly used methods for determining machine-induced forces and other design
loads for foundations supporting machinery. They include definitions and other information on dynamic loads to be requested from the machine manufacturer and alternative assumptions to apply when such data are unavailable or are under-predicted.

3.2.1 Static loads

3.2.1.1 Dead loads—A major function of the foundation is to support gravity (dead) loads due to the weight of the machine, auxiliary equipment, pipe, valves, and deadweight of the foundation structure. The weights of the machine components are normally supplied by the machine manufacturer. The distribution of the weight of the machine on the foundation depends on the location of support points (chocks, soleplates) and on the flexibility of the machine frame. Typically, there are multiple support points, and, thus, the distribution is statically indeterminate. In many cases, the machine manufacturer provides a loading diagram showing the vertical loads at each support point. When this information is not available, it is common to assume the machine frame is rigid and that its weight is appropriately distributed between support points.

3.2.1.2 Live loads—Live loads are produced by personnel, tools, and maintenance equipment and materials. The live loads used in design should be the maximum loads expected during the service life of the machine. For most designs, live loads are uniformly distributed over the floor areas of platforms of elevated support structures or to the access areas around above-grade foundations. Typical live loads vary from 60 lbf/ft² (2.9 kPa) for personnel to as much as 150 lbf/ft² (7.2 kPa) for maintenance equipment and materials.

3.2.1.3 Wind loads—Loads due to wind on the surface areas of the machine, auxiliary equipment, and the support foundation are based on the design wind speed for the particular site and are normally calculated in accordance with the governing local code or standard. Wind loads rarely govern the design of machine foundations except, perhaps, when the machine is located in an enclosure that is also supported by the foundation.

When designing machine foundations and support structures, most practitioners use the wind load provisions of ASCE 7. The analytical procedure of ASCE 7 provides wind pressures and forces for use in the design of the main wind-force resisting systems and anchorage of machine components.

Most structural systems involving machines and machine foundations are relatively stiff (natural frequency in the lateral direction greater than 1 Hz). Consequently, the systems can be treated as rigid with respect to the wind gust effect factor, and simplified procedures can be used. If the machine is supported on flexible isolators and is exposed to the wind, the rigid assumption may not be reasonable, and more elaborate treatment of the gust effects is necessary as described in ASCE 7 for flexible structural systems.

Appropriate consideration of the exposure conditions and importance factors is also required to be consistent with the facilities requirements.

3.2.1.4 Seismic loads—Machinery foundations located in seismically active regions are analyzed for seismic loads. Before 2000, these loads were determined in accordance with methods prescribed in one of various regional building codes (such as the UBC, the SBC, or the NBC) and standards such as ASCE 7 and SEAOC Blue Book.

The publication of the IBC 2000 provides building officials with the opportunity to replace the former regional codes with a code that has nationwide applicability. The seismic requirements in IBC 2000 and ASCE 7-98 are essentially identical, as both are based on the 1997 NEHRP (FEMA 302) provisions.

The IBC and its reference documents contain provisions for design of nonstructural components, including dynamic machinery, for seismic loads. For machinery supported above grade or on more flexible elevated pedestals, seismic amplification factors are also specified.

3.2.1.5 Static operating loads—Static operating loads include the weight of gas or liquid in the machinery equipment during normal operation and forces, such as the drive torque developed by some machines at the connection between the drive mechanism and driven machinery. Static operating loads can also include forces caused by thermal growth of the machinery equipment and connecting piping. Time-varying (dynamic) loads generated by machines during operation are covered elsewhere in this report.

Machines such as compressors and generators require some form of drive mechanism, either integral with the machine or separate from it. When the drive mechanism is nonintegral, such as a separate electric motor, reciprocating engine, and gas or steam turbine, it produces a net external drive torque on the driven machine. The torque is equal in magnitude and opposite in direction on the driver and driven machine. The normal torque (sometimes called drive torque) is generally applied to the foundation as a static force couple in the vertical direction acting about the centerline of the shaft of the machine. The magnitude of the normal torque is often computed from the following formula

\[ NT = \frac{(5250)(P_s)}{f_o} \text{ lbf-ft} \]  
\[ NT = \frac{(9550)(P_s)}{f_o} \text{ N-m} \]

where
\[ NT = \text{normal torque, ft-lbf (m-N)}; \]
\[ P_s = \text{power being transmitted by the shaft at the connection, horsepower (kilowatts)}; \] and
\[ f_o = \text{operating speed, rpm}. \]

The torque load is generally resolved into a vertical force couple by dividing it by the center-to-center distance between longitudinal soleplates or anchor points (Fig. 3.1(a)). When the machine is supported by transverse soleplates only, the torque is applied along the width of the soleplate assuming a straight line variation of force (Fig. 3.1(b)). Normal torque can also be caused by jet forces on turbine blades. In this case it is applied to the foundation in the opposite direction from the rotation of the rotor.

The torque on a generator stator is applied in the same direction as the rotation of the rotor and can be high due to...
startup or an electrical short circuit. Startup torque, a property of electric motors, should be obtained from the motor manufacturer. The torque created by an electrical short circuit is considered a malfunction, emergency, or accidental load and is generally reported separately by the machinery manufacturer. Often in the design for this phenomenon, the magnitude of the emergency drive torque is determined by applying a magnification factor to the normal torque. Consultation with the generator manufacturer is necessary to establish the appropriate magnification factor.

3.2.1.6 Special loads for elevated-type foundations—To ensure adequate strength and deflection control, the following special static loading conditions are recommended in some proprietary standards for large equipment on elevated-type foundations:

1. Vertical force equal to 50% of the total weight of each machine;
2. Horizontal force (in the transverse direction) equal to 25% of the total weight of each machine; and
3. Horizontal force (in the longitudinal direction) equal to 25% of the total weight of each machine.

These forces are additive to normal gravity loads and are considered to act at the centerline of the machine shaft. Loads 1, 2, and 3 are not considered to act concurrently with one another.

3.2.1.7 Erection and maintenance loads—Erection and maintenance loads are temporary loads from equipment, such as cranes and forklifts, required for installing or dismantling machine components during erection or maintenance. Erection loads are usually furnished in the manufacturer’s foundation load drawing and should be used in conjunction with other specified dead, live, and environmental loads. Maintenance loads occur any time the equipment is being drained, cleaned, repaired, and realigned or when the components are being removed or replaced. Loads may result from maintenance equipment, davits, and hoists. Environmental loads, such as full wind and earthquake, are not usually assumed to act with maintenance loads, which generally occur for only a relatively short duration.

3.2.1.8 Thermal loads—Changing temperatures of machines and their foundations cause expansions and contractions, and distortions, causing the various parts to try to slide on the support surfaces. The magnitude of the resulting frictional forces depends on the magnitude of the temperature change, the location of the supports, and on the condition of the support surfaces. The thermal forces do not impose a net force on the foundation to be resisted by soil or piles because the forces on any surface are balanced by equal and opposite forces on other support surfaces. Thermal forces, however, may govern the design of the grout system, pedestals, and hold downs.

Calculation of the exact thermal loading is very difficult because it depends on a number of factors, including distance between anchor points, magnitude of temperature change, the material and condition of the sliding surface, and the magnitude of the vertical load on each soleplate. Lacking a rigorous analysis, the magnitude of the frictional load may be calculated as follows

![Diagram](attachment:image.png)

**a) Torque resisted by longitudinal equipment soleplates**

\[
\text{Force} = \text{Torque} / \text{Spacing}
\]

**b) Torque resisted by transverse equipment soleplates**

\[
\text{Force/Len} = 6 \times \text{Torque} / \text{Width}^2
\]

Fig. 3.1—Equivalent forces for torque loads.

The friction coefficient generally varies from 0.2 to 0.5. Loads acting through the soleplate include: machine dead load, normal torque load, anchor bolt load, and piping loads.

Heat transfer to the foundation can be by convection across an air gap (for example, gap between sump and block) and by conduction through points of physical contact. The resultant temperature gradients induce deformations, strains, and stresses.

When evaluating thermal stress, the calculations are strongly influenced by the stiffness and restraint against deformation for the structural member in question. Therefore, it is important to consider the self-relieving nature of thermal stress due to deformation to prevent being overly conservative in the analysis. As the thermal forces are applied to the foundation member by the machine, the foundation member changes length and thereby provides reduced resistance to the machine forces. This phenomenon can have the effect of reducing the thermal forces from the machine.

Accurate determinations of concrete surface temperatures and thermal gradients are also important. Under steady-state normal operating conditions, temperature distributions across structural sections are usually linear. The air gap between the machine casing and foundation provides a significant means for dissipating heat, and its effect should be included when establishing surface temperatures.

Normally, the expected thermal deflection at various bearings is estimated by the manufacturer, based on past field measurements on existing units. The machine erector then compensates for the thermal deflection during installation.
Reports are available (Mandke and Smalley 1992; Mandke and Smalley 1989; and Smalley 1985) that illustrate the effects of thermal loads and deflections in the concrete foundation of a large reciprocating compressor and their influence on the machine.

### 3.2.2 Rotating machine loads—

Typical heavy rotating machinery include centrifugal air and gas compressors, horizontal and vertical fluid pumps, generators, rotating steam and gas turbine drivers, centrifuges, electric motor drivers, fans, and blowers. These types of machinery are characterized by gas turbine drivers, centrifuges, electric motor drivers, fans, and blowers. These types of machinery are characterized by the rotating motion of one or more impellers or rotors.

#### 3.2.2.1 Dynamic loads due to unbalanced masses—

Unbalanced forces in rotating machines are created when the mass centroid of the rotating part does not coincide with the axis of rotation. In theory, it is possible to precisely balance the rotating elements of rotating machinery. In practice, this is never achieved; slight mass eccentricities always remain. During operation, the eccentric rotating mass produces centrifugal forces that are proportional to the square of machine speed. Centrifugal forces generally increase during the service life of the machine due to conditions such as machine wear, rotor play, and dirt accumulation.

A rotating machine transmits dynamic force to the foundation predominantly through its bearings (with small, generally unimportant exceptions such as seals and the air gap in a motor). The forces acting at the bearings are a function of the level and axial distribution of unbalance, the geometry of the rotor and its bearings, the speed of rotation, and the detailed dynamic characteristics of the rotor-bearing system. At or near a critical speed, the force from rotating unbalance can be substantially amplified, sometimes by a factor of five or more.

Ideally, the determination of the transmitted force under different conditions of unbalance and at different speeds results from a dynamic analysis of the rotor-bearing system, using an appropriate combination of computer programs for calculating bearing dynamic characteristics and the response to unbalance of a flexible rotor in its bearings. Such an analysis would usually be performed by the machine manufacturer. Results of such analyses, especially values for transmitted bearing forces, represent the best source of information for use by the foundation design engineer. This and other approaches used in practice to quantify the magnitude of dynamic force transmitted to the foundation are discussed in Sections 3.2.2.1a to 3.2.2.1c.

#### 3.2.2.1a Dynamic load provided by the manufacturer—

The engineer should request and the machine manufacturer should provide the following information:

**Design levels of unbalance and basis**—This information documents the unbalance level the subsequent transmitted forces are based on.

**Dynamic forces transmitted to the bearing pedestals under the following conditions**—

- a) Under design unbalance levels over operating speed range;
- b) At highest vibration when negotiating critical speeds;
- c) At a vibration level where the machine is just short of tripping on high vibration; and
- d) Under the maximum level of upset condition the machine is designed to survive (for example, loss of one or more blades).

Items a and b document the predicted dynamic forces resulting from levels of unbalance assumed in design for normal operation. Using these forces, it is possible to predict the normal dynamic vibration of the machine on its foundation.

Item c identifies a maximum level of transmitted force with which the machine could operate continuously without tripping; the foundation should have the strength to tolerate such a dynamic force on a continuous basis.

Item d identifies the higher level of dynamic force, which could occur under occasional upset conditions over a short period of time. If the machine is designed to tolerate this level of dynamic force for a short period of time, then the foundation should also be able to tolerate it for a similar period of time.

If an independent dynamic analysis of the rotor-bearing system is performed by the end user or by a third party, such an analysis can provide some or all of the above dynamic forces transmitted to the foundation.

By assuming that the dynamic force transmitted to the bearings equals the rotating unbalanced force generated by the rotor, information on unbalance can provide an estimate of the transmitted force.

#### 3.2.2.1b Machine unbalance provided by the manufacturer—

When the mass unbalance (eccentricity) is known or stated by the manufacturer, the resulting dynamic force amplitude is

\[
F_o = m_r e_m \omega_o^2 S_f / 12 \quad \text{lbf} \tag{3-3}
\]

where

- \(F_o\) = dynamic force amplitude (zero-to-peak), lbf (N);
- \(m_r\) = rotating mass, lbm (kg);
- \(e_m\) = mass eccentricity, in. (mm);
- \(\omega_o\) = circular operating frequency of the machine (rad/s); and
- \(S_f\) = service factor, used to account for increased unbalance during the service life of the machine, generally greater than or equal to 2.

#### 3.2.2.1c Machine unbalance meeting industry criteria—

Many rotating machines are balanced to an initial balance quality in terms of a constant \(e_m / \omega_o\). For example, the normal balance quality \(Q\) for parts of process-plant machinery is 0.25 in./s (6.3 mm/s). Other typical balance quality grade examples are shown in Table 3.1. To meet these criteria a rotor intended for faster speeds should be better balanced than one operating at a slower speed. Using this approach, Eq. (3-3) can be rewritten as

\[
F_o = m_r Q \omega_o S_f / 12 \quad \text{lbf} \tag{3-4}
\]

**3.2.2.1d Results of such analyses, especially values for transmitted bearing forces, represent the best source of information for use by the foundation design engineer.**

---

**Notes:**

- Equations (3-3) and (3-4) are subject to the following conditions:
- Media: air, gas
- Dynamic loads due to unbalanced masses
- Unbalanced forces in rotating machines are created when the mass centroid of the rotating part does not coincide with the axis of rotation. In theory, it is possible to precisely balance the rotating elements of rotating machinery. In practice, this is never achieved; slight mass eccentricities always remain. During operation, the eccentric rotating mass produces centrifugal forces that are proportional to the square of machine speed. Centrifugal forces generally increase during the service life of the machine due to conditions such as machine wear, rotor play, and dirt accumulation.
- A rotating machine transmits dynamic force to the foundation predominantly through its bearings (with small, generally unimportant exceptions such as seals and the air gap in a motor). The forces acting at the bearings are a function of the level and axial distribution of unbalance, the geometry of the rotor and its bearings, the speed of rotation, and the detailed dynamic characteristics of the rotor-bearing system. At or near a critical speed, the force from rotating unbalance can be substantially amplified, sometimes by a factor of five or more.
- Ideally, the determination of the transmitted force under different conditions of unbalance and at different speeds results from a dynamic analysis of the rotor-bearing system, using an appropriate combination of computer programs for calculating bearing dynamic characteristics and the response to unbalance of a flexible rotor in its bearings. Such an analysis would usually be performed by the machine manufacturer. Results of such analyses, especially values for transmitted bearing forces, represent the best source of information for use by the foundation design engineer. This and other approaches used in practice to quantify the magnitude of dynamic force transmitted to the foundation are discussed in Sections 3.2.2.1a to 3.2.2.1c.
- Dynamic load provided by the manufacturer—The engineer should request and the machine manufacturer should provide the following information:
  - **Design levels of unbalance and basis**—This information documents the unbalance level the subsequent transmitted forces are based on.
  - **Dynamic forces transmitted to the bearing pedestals under the following conditions**—
    - a) Under design unbalance levels over operating speed range;
    - b) At highest vibration when negotiating critical speeds;
    - c) At a vibration level where the machine is just short of tripping on high vibration; and
    - d) Under the maximum level of upset condition the machine is designed to survive (for example, loss of one or more blades).
  - Items a and b document the predicted dynamic forces resulting from levels of unbalance assumed in design for normal operation. Using these forces, it is possible to predict the normal dynamic vibration of the machine on its foundation.
  - Item c identifies a maximum level of transmitted force with which the machine could operate continuously without tripping; the foundation should have the strength to tolerate such a dynamic force on a continuous basis.
  - Item d identifies the higher level of dynamic force, which could occur under occasional upset conditions over a short period of time. If the machine is designed to tolerate this level of dynamic force for a short period of time, then the foundation should also be able to tolerate it for a similar period of time.
  - If an independent dynamic analysis of the rotor-bearing system is performed by the end user or by a third party, such an analysis can provide some or all of the above dynamic forces transmitted to the foundation.
  - By assuming that the dynamic force transmitted to the bearings equals the rotating unbalanced force generated by the rotor, information on unbalance can provide an estimate of the transmitted force.
  - **Machine unbalance provided by the manufacturer**—When the mass unbalance (eccentricity) is known or stated by the manufacturer, the resulting dynamic force amplitude is
    \[
    F_o = m_r e_m \omega_o^2 S_f / 12 \quad \text{lbf} \tag{3-3}
    \]
    - where
      - \(F_o\) = dynamic force amplitude (zero-to-peak), lbf (N);
      - \(m_r\) = rotating mass, lbm (kg);
      - \(e_m\) = mass eccentricity, in. (mm);
      - \(\omega_o\) = circular operating frequency of the machine (rad/s); and
      - \(S_f\) = service factor, used to account for increased unbalance during the service life of the machine, generally greater than or equal to 2.
    - **Machine unbalance meeting industry criteria**—Many rotating machines are balanced to an initial balance quality in terms of a constant \(e_m / \omega_o\). For example, the normal balance quality \(Q\) for parts of process-plant machinery is 0.25 in./s (6.3 mm/s). Other typical balance quality grade examples are shown in Table 3.1. To meet these criteria a rotor intended for faster speeds should be better balanced than one operating at a slower speed. Using this approach, Eq. (3-3) can be rewritten as
      \[
      F_o = m_r Q \omega_o S_f / 12 \quad \text{lbf} \tag{3-4}
      \]
Table 3.1—Balance quality grades for selected groups of representative rigid rotors (excerpted from ANSI/ASA S2.19)

<table>
<thead>
<tr>
<th>Balance quality grade</th>
<th>Product of (e_0) in./s (mm/s)</th>
<th>Rotor types—general examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>G100</td>
<td>4 (100)</td>
<td>Crankshaft/drives of fast diesel engines with six or more cylinders</td>
</tr>
<tr>
<td>G40</td>
<td>1.6 (40)</td>
<td>Parts of crushing machines; drive shafts (propeller shafts, cardan shafts) with special requirements; crankshaft/drives of engines with six or more cylinders under special requirements</td>
</tr>
<tr>
<td>G16</td>
<td>0.6 (16)</td>
<td>Parts of process plant machines; centrifuge drums, paper machinery rolls, print rolls; fans; flywheels; pump impellers; machine tool and general machinery parts; medium and large electric armatures (of electric motors having at least 80 mm shaft height) without special requirement</td>
</tr>
<tr>
<td>G6.3</td>
<td>0.25 (6.3)</td>
<td>Gas and steam turbines, including marine main turbines; rigid turbo-generator rotors; turbo-compressors; machine tool drives; medium and large electric armatures with special requirements; turbine driven pumps</td>
</tr>
<tr>
<td>G2.5</td>
<td>0.1 (2.5)</td>
<td>Grinding machine drives</td>
</tr>
<tr>
<td>G1</td>
<td>0.04 (1)</td>
<td>Spindles, discs, and armatures of precision grinders</td>
</tr>
<tr>
<td>G0.4</td>
<td>0.015 (0.4)</td>
<td></td>
</tr>
</tbody>
</table>

API 617 and API 684 work with maximum residual unbalance \(U_{\text{max}}\) criteria for petroleum processing applications. The mass eccentricity is determined by dividing \(U_{\text{max}}\) by the rotor weight. For axial and centrifugal compressors with maximum continuous operating speeds greater than 25,000 rpm, API 617 establishes a maximum allowable mass eccentricity of \(10 \times 10^{-6}\) in. (250 nm). For compressors operating at slower speeds, the maximum allowable mass eccentricity is

\[
e_m = 0.25 f_o \text{ in.} \tag{3-5}
\]

\[
e_m = 6.35 f_o \text{ mm} \tag{3-5}
\]

where

\(f_o = \text{operating speed, rpm} \leq 25,000 \text{ rpm}.\)

This permitted initial mass eccentricity is tighter than ISO balance quality grade G2.5, which would be applied to this type of equipment (Table 3.1, turbo-compressors) under ISO 1940. As such, the dynamic force computed from this API consideration will be quite small and a larger service factor might be used to have a realistic design force.

API 617 also identifies a limitation on the peak-to-peak vibration amplitude during mechanical testing of the compressor with the equipment operating at its maximum continuous speed \((12,000/f_o)^{0.5} \text{ in.} \leq 25.4(12,000/f_o)^{0.5} \text{ mm}.\)

Some design firms use this criterion and a service factor \(S_f\) of 2.0 to compute the dynamic force amplitude as

Equations (3-3), (3-4), (3-6), and (3-7) appear to be very different: the exponents on the speed of rotation vary from 1 to 1.5 to 2, constants vary widely, and different variables appear. Some equations use mass, others use weight. In reality, the equations are more similar than they appear. Given the right understanding of \(Q\) as a replacement for \(e_0\), Eq. (3-3), (3-4), and (3-7) take on the same character. These equations then indicate that the design force at operating speed varies linearly with both the mass of the rotating body and the operating rotational speed. Once that state is identified, Eq. (3-3) can be adjusted to reflect the actual speed of rotation, and the dynamic centrifugal force is seen to vary with the square of the speed. Restating Eq. (3-6) and (3-7) in the form of Eq. (3-3) allows for the development of an effective eccentricity implied within these equations with the comparison shown in Fig. 3.2. Equation (3-7) produces the same result as Eq. (3-4) using \(Q = 0.25 \text{ in./s} (6.3 \text{ mm/s})\), and \(S_f = 2.5\).

The centrifugal forces due to mass unbalance are considered to act at the center of gravity of the rotating part and vary harmonically at the speed of the machine in the two orthogonal directions perpendicular to the shaft. The forces in the two orthogonal directions are equal in magnitude and 90 degrees out of phase and are transmitted to the foundation through the
bearings. Schenck (1990) provides useful information about balance quality for various classes of machinery.

3.2.2.1e Machine unbalance determined from trip vibration level and effective bearing stiffness—Because a rotor is often set to trip on high vibration, it can be expected to operate continuously at any vibration level up to the trip limit. Given the effective bearing stiffness, it is possible to calculate the maximum dynamic force amplitude as

\[ F_0 = V_{\text{max}} K_{\text{eff}} \]  

(3-8)

where

\( V_{\text{max}} \) = the maximum allowable vibration, in. (mm); and

\( K_{\text{eff}} \) = the effective bearing stiffness, lbf/in. (N/mm).

To use this approach, the manufacturer should provide effective bearing stiffness or the engineer should calculate it from the bearing geometry and operating conditions (such as viscosity and speed).

3.2.2.2 Loads from multiple rotating machines—If a foundation supports multiple rotating machines, the engineer should compute unbalanced force based on the mass, unbalance, and operating speed of each rotating component. The response to each rotating mass is then combined to determine the total response. Some practitioners, depending on the specific situation of machine size and criticality, find it advantageous to combine the unbalanced forces from each rotating component into a single resultant unbalanced force. The method of combining two dynamic forces is up to individual judgment and often involves some approximations. In some cases, loads or responses can be added absolutely. In other cases, the loads are treated as out-of-phase so that twisting effects are increased. Often, the operating speed of the equipment should be considered. Even if operating speeds are nominally the same, the design engineer should recognize that during normal operation, the speed of the machines will vary and beating effects can develop. Beating effects develop as two machines operate at close to the same speed. At one point in time, responses to the two machines are additive and motions are maximized. A short time later, the responses cancel each other and the motions are minimized. The net effect is a continual cyclic rising and falling of motion.

3.2.3 Reciprocating machine loads—Internal-combustion engines, piston-type compressors and pumps, some metal forming presses, steam engines, and other machinery are characterized by the rotating motion of a master crankshaft and the linear reciprocating motion of connected pistons or sliders. The motion of these components cause cyclically varying forces, often called reciprocating forces.

3.2.3.1 Primary and secondary reciprocating loads—The simplest type of reciprocating machine uses a single crank mechanism as shown in Fig. 3.3. The idealization of this mechanism consists of a piston that moves within a guiding cylinder, a crank of length \( r \) that rotates about a crank shaft, and a connecting rod of length \( L \). The connecting rod is attached to the piston at point \( P \) and to the crank at point \( C \). The wrist pin \( P \) oscillates while the crank pin \( C \) follows a circular path. This idealized single cylinder illustrates the concept of a machine producing both primary and secondary reciprocating forces.

If the crank is assumed to rotate at a constant angular velocity \( \omega_0 \), the translational acceleration of the piston along its axis may be evaluated. If \( Z_p \) is defined as the piston displacement toward the crankshaft (local Z-axis), an expression can be written for \( Z_p \) at any time \( t \). Further, the velocity and acceleration can also be obtained by taking the first and second derivatives of the displacement expression with respect to time. The displacement, velocity, and acceleration expressions for the motion of the piston are as follows

\[ Z_p = \left( r + \frac{r^2}{4L} \right) - r \left( \cos \omega_0 t + \frac{r}{4L} \cos 2\omega_0 t \right) \]  

(3-9)

\[ \dot{Z}_p = r \omega_0 \left( \sin \omega_0 t + \frac{r}{2L} \sin 2\omega_0 t \right) \]  

(3-10)

\[ \ddot{Z}_p = r \omega_0^2 \left( \cos \omega_0 t \frac{r^2}{L} \cos 2\omega_0 t \right) \]  

(3-11)

where

\( Z_p \) = piston displacement, in. (mm);

\( r \) = length of crank, in. (mm);

\( L \) = length of connecting rod, in. (mm);

\( \omega_0 \) = circular operating frequency of the machine (rad/s); and

\( t \) = time, s.

Note that the expressions contain two terms each with a sine or cosine; the term that varies with the frequency of the rotation, \( \omega_0 \), is referred to as the primary term while the term that varies at twice the frequency of rotation, \( 2\omega_0 \), is called the secondary term.

Similar expressions can be developed for the local Z-axis (parallel to piston movement) and local Y-axis (perpendicular to piston movement) motion of the rotating parts of the crank. If any unbalance in the crankshaft is replaced by a mass concentrated at the crank pin \( C \), such that the inertia forces are the same as in the original system, the following terms for motion at point \( C \) can be written
\[ y_p = -r \sin \omega_o t \quad \text{(3-12)} \]
\[ \dot{y}_p = -r \omega \cos \omega_o t \quad \text{(3-13)} \]
\[ \ddot{y}_p = r \omega^2 \sin \omega_o t \quad \text{(3-14)} \]
\[ z_p = r (1 - \cos \omega_o t) \quad \text{(3-15)} \]
\[ \dot{z}_p = r \omega \sin \omega_o t \quad \text{(3-16)} \]
\[ \ddot{z}_p = r \omega^2 \cos \omega_o t \quad \text{(3-17)} \]

where
\[ y_p \] = crank pin displacement in local Y-axis, in. (mm); and
\[ z_p \] = crank pin displacement in local Z-axis, in. (mm).

Identifying a part of the connecting rod (usually 1/3 of its mass) plus the piston as the reciprocating mass \( m_{rec} \) concentrated at point P and designating the remainder of the connecting rod plus the crank as the rotating mass \( m_{rot} \) concentrated at point C, expressions for the unbalanced forces are as follows.

Parallel to piston movement
\[ F_z = (m_{rec} + m_{rot}) r \omega^2 \cos \omega_o t + m_{rec} \frac{r \omega^2}{L} \cos 2 \omega_o t \quad \text{(3-18)} \]

Perpendicular to piston movement
\[ F_Y = m_{rot} r \omega^2 \sin \omega_o t \quad \text{(3-19)} \]

Note that Eq. (3.18) consists of two terms, a primary force
\[ (m_{rec} + m_{rot}) r \omega^2 \cos \omega_o t \quad \text{(3-20)} \]
and a secondary force
\[ m_{rec} \frac{r \omega^2}{L} \cos 2 \omega_o t \quad \text{(3-21)} \]

whereas Eq. (3.19) has only a primary component.

### 3.2.3.2 Compressor gas loads

A reciprocating compressor raises the pressure of a certain flow of gas by imparting reciprocating motion on a piston within a cylinder. The piston normally compresses gas during both directions of reciprocating motion. As gas flows to and from each end, the pressure of the gas increases as it is compressed by each stroke of the piston. The increase in pressure within the cylinder creates reaction forces on the head and crank ends of the piston which alternate as gas flows to and from each end of the cylinder.

The gas force contributed to the piston rod equals the instantaneous difference between the pressure force acting on the head and crank end of the piston as shown in Fig. 3.4.

The following formulation can be used to estimate the maximum force acting on the piston rod of an individual double-acting cylinder
\[ F_{rod} = \left[ (P_{head})(A_{head}) - (P_{crank})(A_{crank}) \right] F_1 \quad \text{(3-22)} \]
\[ A_{head} = \frac{\pi}{4} B_c^2 \quad \text{(3-23)} \]
\[ A_{crank} = \frac{\pi}{4} (B_c^2 - D_{rod}^2) \quad \text{(3-24)} \]

where
\[ F_{rod} \] = force acting on piston rod, lbf (N);
\[ A_{head} \] = head and crank areas, in.² (mm²);
\[ B_c \] = cylinder bore diameter, in. (mm);
\[ D_{rod} \] = rod diameter, in. (mm);
\[ P_{head} \] = instantaneous head and crank pressures, psi, (MPa); and
\[ F_1 \] = correction factor.

The head and crank end pressures vary continuously and the differential force takes both positive and negative net values during each cycle of piston motion. The normal approach is to establish the head and crank pressures using the maximum and minimum suction and discharge pressures. For design purposes, it is common to multiply Eq. (3.22) by a factor \( F_1 \) to help account for the natural tendency of gas forces to exceed the values based directly on suction and discharge pressures due to flow resistances and pulsations. Machines with good pulsation control and low external flow resistance may achieve \( F_1 \) as small as 1.1; for machines with low compression ratio, high pulsations, or highly resistive flow through piping and nozzles, \( F_1 \) can approach 1.5 or even higher. A reasonable working value for \( F_1 \) is 1.15 to 1.2.

Preferably, the maximum rod force resulting from gas pressures is based on knowledge of the continuous variation of pressure in the cylinder (measured or predicted). In a repair situation, measured cylinder pressure variation using a cylinder analyzer provides the most accurate value of gas forces. Even without cylinder pressure analysis, extreme operating values of
suction and discharge pressure for each stage should be recorded before the repair and used in the Eq. (3-22).

On new compressors, the engineer should ask the machine manufacturer to provide values for maximum compressive and tensile gas loads on each cylinder rod and, if these are based on suction and discharge pressures, to recommend a value of $F_1$.

Gas forces act on the crankshaft with an equal and opposite reaction on the cylinder. Thus, crankshaft and cylinder forces globally balance each other. Between the crankshaft and the cylinder, however, the compressor frame stretches or contracts in tension or compression under the action of the gas forces. The forces due to frame deflections are transmitted to the foundation through connections with the compressor frame. When acting without slippage, the frame and foundation become an integral structure and together stretch or contract under the gas loads.

The magnitude of gas force transferred into the foundation depends on the relative flexibility of the compressor frame. A very stiff frame transmits only a small fraction of the gas force while a very flexible frame transmits most or all of the force. Similar comments apply to the transfer of individual cylinder inertia forces.

Based on limited comparisons using finite element analysis (Smalley 1988), the following guideline is suggested for gas and inertia force loads transmitted to the foundation by a typical compressor

$$F_{\text{block}} = F_{\text{rod}}/F_{\text{red}} \quad (3-25)$$

$$F_{\text{bolt}}_{\text{CHG}} = [F_{\text{rod}}/\{N_{\text{bolt}}_{\text{CHG}}\}]F_{\text{red}} \quad (3-26)$$

$$F_{\text{bolt}}_{\text{frame}} = [F_{\text{unbalance}}/\{N_{\text{bolt}}\}]F_{\text{red}} \quad (3-27)$$

where

- $F_{\text{block}}$ = the force acting outward on the block from which concrete stresses should be calculated, lbf (N);
- $F_{\text{bolt}}_{\text{CHG}}$ = the force to be restrained by friction at the cross head guide tie-down bolts, lbf (N);
- $F_{\text{bolt}}_{\text{frame}}$ = the force to be restrained by friction at the frame tie-down bolts, lbf (N);
- $F_{\text{red}}$ = a force reduction factor with suggested value of 2, to account for the fraction of individual cylinder load carried by the compressor frame (“frame rigidity factor”);
- $N_{\text{bolt}}_{\text{CHG}}$ = the number of bolts holding down one crosshead guide;
- $N_{\text{bolt}}_{\text{frame}}$ = the number of bolts holding down the frame, per cylinder;
- $F_{\text{rod}}$ = force acting on piston rod, from Eq. (3-22), lbf (N); and
- $F_{\text{unbalance}}$ = the maximum value from Eq. (3-18) applied using parameters for a horizontal compressor cylinder, lbf (N).

The factor $F_{\text{red}}$ is used to simplify a complex problem, thus avoiding the application of unrealistically high loads on the anchor bolts and the foundation block. The mechanics involved in transmitting loads are complex and cannot easily be reduced to a simple relationship between a few parameters beyond the given load equations. A detailed finite-element analysis of metal compressor frame, chock mounts, concrete block, and grout will account for the relative flexibility of the frame and its foundation in determining individual anchor bolt loads and implicitly provide a value for $F_{\text{red}}$. If the frame is very stiff relative to the foundation, the value for $F_{\text{red}}$ will be higher, implying more of the transmitted loads are carried by the frame and less by the anchor bolts and foundation block. Based on experience, a value of 2 for this factor is conservatively low; however, higher values have been seen with frames designed to be especially stiff.

Simplifying this approach, one report (Smalley and Harrell 1997) suggests using a finite element analysis to calculate forces transmitted to the anchor bolts. If a finite element analysis is not possible, the engineer should get from the machine manufacturer or calculate the maximum horizontal gas force and maximum horizontal inertia force for any throw or cylinder. The mounts, anchor bolts, and blocks are then designed for

$$F_{\text{throw}} = (\text{greater of } F_{\text{GMAX}} \text{ or } F_{\text{IMAX}})/2 \quad (3-28)$$

where

- $F_{\text{GMAX}}$ = maximum horizontal gas force on a throw or cylinder, lbf (N);
- $F_{\text{IMAX}}$ = maximum horizontal inertia force on a throw or cylinder, lbf (N); and
- $F_{\text{throw}}$ = horizontal force to be resisted by each throw’s anchor bolts, lbf (N).

3.2.3.3 Reciprocating inertia loads for multicylinder machines—As a practical matter, most reciprocating machines have more than one cylinder, and manufacturers arrange the machine components in a manner that minimizes the net unbalanced forces. For example, rotating parts like the crankshaft can be balanced by adding or removing correcting weights. Translating parts like pistons and those that exhibit both rotation and translation, like connecting rods, can be arranged in such a way as to minimize the unbalanced forces and moments generated. Seldom, if ever, is it possible to perfectly balance reciprocating machines.

The forces generated by reciprocating mechanisms are functions of the mass, stroke, piston arrangement, connecting rod size, crank throw orientation (phase angle), and the mass and arrangement of counterweights on the crankshaft. For this reason, calculating the reciprocating forces for multicylinder machines can be quite complex and are therefore normally provided by the machine manufacturer. If the machine is an integral engine compressor, it can include, in one frame, cylinders oriented horizontally, vertically, or in between, all with reciprocating inertias.

Some machine manufacturers place displacement transducers and accelerometers on strategic points on the machinery. They can then measure displacements and accelerations at those points for several operational frequencies to determine the
magnitude of the unbalanced forces and couples for multicylinder machines.

3.2.3.4 Estimating reciprocating inertia forces from multicylinder machines—In cases where the manufacturer’s data are unavailable or components are being replaced, the engineer should use hand calculations to estimate the reciprocating forces from a multicylinder machine. One such procedure for a machine having n number of cylinders is discussed by Mandke and Troxler (1992). Section 3.7 summarizes this method.

3.2.4 Impulsive machine loads—The impulsive load generated by a forging hammer is caused by the impact of the hammer ram onto the hammer anvil. This impact process transfers the kinetic energy of the ram into kinetic energy of the entire hammer assembly. The post-impact velocity of the hammer is represented by

\[ v_h = \frac{M_r}{M_h}(1 + \alpha_h)v_r \]  

(3-29)

where
\[ v_h = \text{post-impact hammer velocity, ft/s (m/s)}; \]
\[ M_r = \text{ram mass including dies and ancillary parts, lbm (kg)}; \]
\[ M_h = \text{hammer mass including any auxiliary foundation, lbm (kg)}; \]
\[ \alpha_h = \text{ram rebound velocity relative to impact velocity; and} \]
\[ v_r = \text{ram impact velocity, ft/s (m/s)}; \]

General experience indicates that \( \alpha_h \) is approximately 60% for many forging hammer installations. From that point, the hammer foundation performance can be assessed as a rigid body oscillating as a single degree-of-freedom system with an initial velocity of \( v_r \).

For metal-forming presses, the dynamic forces develop from two sources: the mechanical movement of the press components and material-forming process. Each of these forces is unique to the press design and application and needs to be evaluated with proper information from the press manufacturer and the owner.

The press mechanics often include rotating and reciprocating components. The dynamic forces from these individual pieces follow the rules established in earlier sections of this document for rotating and reciprocating components. Only the press manufacturer familiar with all the internal components can knowledgeably calculate the specific forces. Figure 2.3 presents a horizontal force time-history for a forging press. Similar presses can be expected to have similar characteristics; however, the particular values and timing data differ.

The press drive mechanisms include geared and direct-drive systems. Depending on the design, these drives may or may not be balanced. The press slide travels vertically through a set stroke of 1/2 in. (12 mm) to several inches at a given speed. Some small presses may have inclinable beds so that the slide is not moving vertically. It is often adequate to assume that the slide moves in a vertical path defined by a circularly rotating crankshaft, that is

\[ d_s(t) = \frac{S}{2} \sin(\omega_o t) \]  

(3-30)

where
\[ d_s = \text{displacement of the slide, in. (mm)}; \]
\[ S = \text{press stroke, in. (mm)}; \] and
\[ \omega_o = \text{circular operating frequency of the machine (rad/s)}. \]

This leads to a dynamic inertia force from the slide of

\[ F_s(t) = m_d\omega_o^2S\sin(\omega_o t)/12 \text{ lbf} \]  

(3-31)

\[ F_s(t) = m_d\omega_o^2S\sin(\omega_o t)/1000 \text{ N} \]

where
\[ F_s = \text{dynamic inertia force of slide, lbf (N)}; \] and
\[ m_d = \text{slide mass including the effects of any balance mechanism, lbm (kg)}. \]

This assumption is based on simple circular motions and simple linkages. Other systems may be in-place to increase the press force and improve the timing. These other systems may increase the acceleration of the unbalanced weights and thus alter the magnitude and frequency components of the dynamic force transmitted to the foundation.

3.2.5 Loading conditions—During their lives, machinery equipment support structures and foundations undergo different loading conditions including erection, testing, shut-down, maintenance, and normal and abnormal operation. For each loading condition, there can be one or more combinations of loads that apply to the structure or foundation. The following loading conditions are generally considered in design:

- Erection condition represents the design loads that act on the structure/foundation during its construction;
- Testing condition represents the design loads that act on the structure/foundation while the equipment being supported is undergoing testing, such as hydrotest;
- Empty (shutdown) represents the design loads that act on the structure when the supported equipment is at its least weight due to removal of process fluids, applicable internals, or both as a result of maintenance or other out-of-service disruption;
- Normal operating condition represents the design loading during periods of normal equipment operation;
- Abnormal operating condition represents the design loading during periods when unusual or extreme operating loads act on the structure/foundation.

3.2.6 Load combinations—Table 3.2 shows the general classification of loads for use in determining the applicable load factors in strength design (ACI 318). In considering soil stresses, the normal approach is working stress design without load factors and with overall factors of safety identified as appropriate by geotechnical engineers. The load combinations frequently used for the various load conditions are as follows:

1. Erection
   a) Dead load + erection forces
Table 3.2—Load classifications for ultimate strength design

<table>
<thead>
<tr>
<th>Design loads</th>
<th>Load classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of structure, equipment, internals, insulation, and platforms</td>
<td>Dead</td>
</tr>
<tr>
<td>Temporary loads and forces caused by erection</td>
<td></td>
</tr>
<tr>
<td>Fluid loads during testing and operation</td>
<td></td>
</tr>
<tr>
<td>Thermal loads</td>
<td></td>
</tr>
<tr>
<td>Anchor and guide loads</td>
<td></td>
</tr>
<tr>
<td>Platform and walkway loads</td>
<td>Live</td>
</tr>
<tr>
<td>Materials to be temporarily stored during maintenance</td>
<td></td>
</tr>
<tr>
<td>Materials normally stored during operation such as tools and maintenance equipment</td>
<td></td>
</tr>
<tr>
<td>Vibrating equipment forces</td>
<td></td>
</tr>
<tr>
<td>Impact loads for hoist and equipment handling utilities</td>
<td></td>
</tr>
<tr>
<td>Earthquake loads</td>
<td>Environmental</td>
</tr>
<tr>
<td>Transportation loads</td>
<td></td>
</tr>
<tr>
<td>Snow, ice, or rain loads</td>
<td></td>
</tr>
<tr>
<td>Wind loads</td>
<td></td>
</tr>
</tbody>
</table>

b) Dead load + erection forces + reduced wind + snow, ice, or rain

c) Dead load + erection forces + seismic + snow, ice, or rain

2. Testing

a) Dead load + test loads
b) Dead load + test loads + live + snow, ice, or rain
c) Dead load + test loads + reduced wind + snow, ice, or rain

3. Empty (shutdown)

a) Dead load + maintenance forces + live load + snow, ice, or rain

4. Normal operation

a) Dead load
b) Dead load + thermal load + machine forces + live loads + wind + snow, ice, or rain
c) Dead load + thermal load + machine forces + seismic + snow, ice, or rain

5. Abnormal operation

a) Dead load + upset (abnormal) machine loads + live + reduced wind

It is common to only use some fraction of full wind, such as 80% in combination with erection loads and 33% for test loads, due to the short duration of these conditions (ASCE 7).

3.3—Dynamic soil properties

Soil dynamics deals with engineering properties and behavior of soil under dynamic stress. For the dynamic analysis of machine foundations, soil properties, such as Poisson’s ratio, dynamic shear modulus, and damping of soil, are generally required.

Though this work is typically completed by a geotechnical engineer, this section provides a general overview of methods used to determine the various soil properties. Many references are available that provide a greater level of detail on both theory and standard practice, including Das (1993), Bowles (1996), Fang (1991), and Arya, O’Neill, and Pincus (1979). Seed and Idriss (1970) provide greater detail on items that influence different soil properties.

This section does not cover considerations that affect the suitability of a given soil to support a dynamic machine foundation. Problems could include excessive settlement caused by dynamic or static loads, liquefaction, dimensional stability of a cohesive soil, frost heave, or any other relevant soils concern.

In general, problems involving the dynamic properties of soils are divided into small and large strain amplitude responses. For machine foundations, the amplitudes of dynamic motion, and consequently the strains in the soil, are usually low (strains less than $10^{-3}\%$). A foundation that is subjected to an earthquake or blast loading is likely to undergo large deformations and, therefore, induce large strains in the soil. The information in this report is only applicable for typical machine foundation strains. Refer to Seed and Idriss (1970) for information on strain-related effects on shear modulus and material damping.

The key soil properties, Poisson’s ratio and dynamic shear modulus, may be significantly affected by water table variations. Prudence suggests that in determining these properties, such variations be considered and assessed, usually in conjunction with the geotechnical engineers. This approach often results in expanding the range of properties to be considered in the design phase.

3.3.1 Poisson’s ratio—Poisson’s ratio, which is the ratio of the strain in the direction perpendicular to loading to the strain in the direction of loading, is used to calculate both the soil stiffness and damping. Poisson’s ratio can be computed from the measured values of wave velocities traveling through the soil. These computations, however, are difficult. The stiffness and damping of a foundation system are generally insensitive to variations of Poisson’s ratio common in soils.

Generally, Poisson’s ratio varies from 0.25 to 0.35 for cohesionless soils and from 0.35 to 0.45 for cohesive soils. If no specific values of Poisson’s ratio are available, then, for design purposes, the engineer may take Poisson’s ratio as 0.33 for cohesionless soils and 0.40 for cohesive soils.

3.3.2 Dynamic shear modulus—Dynamic shear modulus $G$ is the most important soil parameter influencing the dynamic behavior of the soil-foundation system. Together with Poisson’s ratio, it is used to calculate soil impedance. Refer to Section 4.2 for the discussion on soil impedance.

The dynamic shear modulus represents the slope of the shear stress versus shear strain curve. Most soils do not respond elastically to shear strains; they respond with a combination of elastic and plastic strain. For that reason, plotting shear stress versus shear strain results in a curve not a straight line. The value of $G$ varies based on the strain considered. The lower the strain, the higher the dynamic shear modulus.

Several methods are available for obtaining useful values of dynamic shear modulus:

- Field measurements of stress wave velocities of in-place soils;
- Laboratory tests on soil samples; and
- Correlation to other soil properties.

Due to variations inherent in the determination of dynamic shear modulus values, it may be appropriate to complete more than one foundation analysis. One analysis could be completed with the minimum possible value, one could be completed using the maximum possible value, and then additional analyses could be completed with intermediate values.
3.3.2.1 Field determination—Field measurements are the most common method for determining the dynamic shear modulus of a given soil. These methods involve measuring the soil characteristics, in-place, as close as possible to the actual foundation location(s).

Because field determinations are an indirect determination of shear modulus, the specific property measured is the shear wave velocity. There are three different types of stress waves that can be transmitted through soil or any other elastic body.

- Compression (primary P) waves;
- Shear (secondary S) waves; and
- Rayleigh (surface) waves.

Compression waves are transmitted through soil by a volume change associated with compressive and tensile stresses. Compression waves are the fastest of the three stress waves.

Shear waves are transmitted through soil by distortion associated with shear stresses in the soil and are slower than compression waves. No volume change occurs in the soil. Rayleigh waves occur at the free surface of an elastic body; typically, this is the ground surface. Rayleigh waves have components that are both perpendicular to the free surface and parallel to the free surface and are slightly slower than shear waves.

Several methods are available for measuring wave velocities of the in-place soil:

- The cross-hole method;
- The down-hole method;
- The up-hole method; and
- Seismic reflection (or refraction).

In the cross-hole method, two vertical boreholes are drilled. A signal generator is placed in one hole and a sensor is placed in the other hole. An impulse signal is generated in one hole, and then the time the shear wave takes to travel from the signal generator to the sensor is measured. The travel time divided by the distance yields the shear wave velocity. The cross-hole method can be used to determine $G$ at different depths (Fig. 3.5).

In the down-hole method, only one vertical borehole is drilled. A signal generator is placed at the ground surface some distance away from the borehole, and a sensor is placed in the bottom of the borehole. An impulse signal is generated, and then the time the shear wave takes to travel from the signal generator to the sensor is measured. The travel time divided by the distance yields the shear wave velocity. This method can be run several different times, with the signal generator located at different distances from the borehole each time. This permits the measuring of soil properties at several different locations, which can then be averaged to determine an average shear wave velocity (Fig. 3.6).

The up-hole method is similar to the down-hole method. The difference is that the signal generator is placed in the borehole and the sensor is placed at the ground surface.

Dynamic shear modulus and measured-in-field shear wave velocity are related as follows

$$ G = \rho (V_s)^2 $$

where

- $G$ = dynamic shear modulus of the soil, lbf/ft$^2$ (Pa);
- $V_s$ = shear wave velocity of the soil, ft/s (m/s); and
- $\rho$ = soil mass density, lbm/ft$^3$ (kg/m$^3$).

An alternative field method is to use reflection or refraction of elastic stress waves. These methods are based on the principle that when elastic waves hit a boundary between dissimilar layers, the wave is reflected or refracted. This method should only be used at locations where the soils are deposited in discrete horizontal, or nearly horizontal layers, or at locations where soil exists over top of bedrock. This method consists of generating a stress wave at one location at the ground surface and measuring the time it takes for the stress wave to reach a second location at the ground surface. The wave travels from the ground surface to the interface between differing soils layers, travels along the interface, then back to the ground surface. The time the wave takes to travel from the signal generator to the sensor is a function of the soils properties and the depth of the soil interface. One advantage of this method is that no boreholes are required. Also, this method yields an estimated depth to differing soil layers. One disadvantage is that this method cannot be used when the groundwater table is near the ground surface.

3.3.2.2 Laboratory determination—Laboratory tests are considered less accurate than field measurements due to the
Table 3.3—Values of $K_2$ versus relative density (Seed and Idriss 1970)

<table>
<thead>
<tr>
<th>Relative density, %</th>
<th>$K_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>70</td>
</tr>
<tr>
<td>75</td>
<td>61</td>
</tr>
<tr>
<td>60</td>
<td>52</td>
</tr>
<tr>
<td>45</td>
<td>43</td>
</tr>
<tr>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>35</td>
<td>34</td>
</tr>
</tbody>
</table>

possibility of sample disturbance. Sometimes laboratory tests are used to validate field measurements when a high level of scrutiny is required, for instance, when soil properties are required for a nuclear energy facility.

The most common laboratory test is the Resonant-Column method, where a cylindrical sample of soil is placed in a device capable of generating forced vibrations. The soil sample is exited at different frequencies until the resonant frequency is determined. The dynamic soil modulus can be calculated based on the frequency, the length of the soil sample, the end conditions of the soil sample, and the density of the soil sample. ASTM D 4015 defines the Resonant-Column method.

3.3.2.3 Correlation to other soil properties—Correlation is another method for determining dynamic soils properties. The engineer should be careful when using any correlation method because these are generally the least-accurate methods. The most appropriate time to consider using these methods is for preliminary design or for small noncritical applications with small dynamic loads. Correlation to other soil properties should be considered as providing a range of possible values, not providing a single exact value.

Hardin and Richart (1963) determined that soil void ratio $e_v$ and the probable confining pressure $\sigma_0$ had the most impact on the dynamic shear modulus. Hardin and Black (1968) developed the following relationships:

For round-grained sands with $e < 0.8$, dynamic shear modulus can be estimated from

$$G = \frac{31,530(2.17 - e_v)^2 \sqrt{\sigma_0}}{1 + e_v} \text{ lbf/ft}^2 \quad (3-33)$$

$$G = \frac{218,200(2.17 - e_v)^2 \sqrt{\sigma_0}}{1 + e_v} \text{ Pa} \quad (3-34)$$

For angular-grained materials with $e > 0.6$ and normally consolidated clays with low surface activity, dynamic shear modulus can be estimated from

$$G = \frac{14,760(2.97 - e_v)^2 \sqrt{\sigma_0}}{1 + e_v} \text{ lbf/ft}^2$$

$$G = \frac{102,140(2.97 - e_v)^2 \sqrt{\sigma_0}}{1 + e_v} \text{ Pa}$$

In the previous equations, $e_v = \text{void ratio}$; and $\sigma_0 = \text{probable confining pressure, lbf/ft}^2$ (Pa).

In general, relative density in sand is proportional to the void ratio. Seed and Idriss (1970) provide guidance for correlating the dynamic shear modulus to relative density in sand, along with the confining pressure

$$G = 1000K_2\sqrt{\sigma_0} \text{ lbf/ft}^2$$

$$G = 6920K_2\sqrt{\sigma_0} \text{ Pa} \quad (3-35)$$

where $K_2$ is a parameter that depends on void ratio and strain amplitude. Table 3.3 provides values of $K_2$ with respect to relative density.

3.3.3 Damping of soil—Damping is a phenomenon of energy dissipation that opposes free vibrations of a system. Like the restoring forces, the damping forces oppose the motion, but the energy dissipated through damping cannot be recovered. A characteristic feature of damping forces is that they lag the displacement and are out of phase with the motion. Damping of soil includes two effects—geometric and material damping.

Geometric, or radiation, damping reflects energy dissipation through propagation of elastic waves away from the immediate vicinity of a foundation and inelastic deformation of soil. It results from the practical infinity of the soil medium, and it is close to viscous in character. Refer to Chapter 4 for methods of computing geometric damping.

Material, or hysteretic, damping reflects energy dissipation within the soil itself due to the imperfect elasticity of real materials, which exhibit a hysteric loop effect under cyclic loading (Fig. 3.7). The amount of dissipated energy is given by the area of the hysteretic loop. The hysteretic loop implies a phase shift between the stress and strain because there is a stress at zero strain and vice versa, as can be seen from Fig. 3.7. The amount of dissipated energy depends on strain (displacement) but is essentially independent of frequency, as shown on Fig. 3.8.

The magnitude of material damping can be established experimentally using the hysteretic loop and the relation

$$\beta_m = \frac{1}{4\pi} \frac{\Delta W}{W} \quad (3-36)$$

where

$\beta_m = \text{material damping ratio}$;

$\Delta W = \text{area enclosed by the hysteretic loop}$; and

$W = \text{strain energy}$.

Instead of an experimental determination, many practitioners use a material damping ratio of 0.05, or 5%. The material damping ratio is fairly constant for small strains but increases with strain due to the nonlinear behavior of soils.

The term material or hysteretic damping implies frequency independent damping. Experiments indicate that frequency independent hysteretic damping is much more
3.4—Vibration performance criteria

The main purposes of the foundation system with respect to dynamic loads include limiting vibrations, internal loads, and stresses within the equipment. The foundation system also limits vibrations in the areas around the equipment where other vibration-sensitive equipment may be installed, personnel may have to work on a regular basis, or damage to the surrounding structures may occur. These performance criteria are usually established based on vibration amplitudes at key points on or around the equipment and foundation system. These amplitudes may be based on displacement, velocity, or acceleration units. Displacement limitations are commonly based on peak-to-peak amplitudes measured in mils (0.001 in.) or microns (10^-6 m). Velocity limitations are typically based on either peak velocities or root-mean-square (rms) velocities in units of inch per second or millimeter per second. Displacement criteria are almost always frequency dependent with greater motions tolerated at slower speeds. Velocity criteria may depend on frequency but are often independent. Acceleration criteria may be constant with frequency or dependent.

Some types of equipment operate at a constant speed while other types operate across a range of speeds. The foundation engineer should consider the effect of these speed variations during the foundation design.

3.4.1 Machine limits—The vibration limits applicable to the machine are normally set by the equipment manufacturer or are specified by the equipment operator or owner. The limits are usually predicated on either limiting damage to the equipment or ensuring proper performance of the equipment. Limits specified by operators of the machinery and design engineers are usually based on such factors as experience or the installation of additional vibration monitoring equipment.

For rotating equipment (fans, pumps, and turbines), the normal criterion limits vibration displacements or velocities at the bearings of the rotating shaft. Excessive vibrations of the bearings increase maintenance requirements and lead to premature failure of the bearings. Often, rotating equipment has vibration switches to stop the equipment if vibrations become excessive.

Reciprocating equipment (diesel generators, compressors, and similar machinery) tends to be more dynamically rugged than rotating equipment. At the same time, it often generates greater dynamic forces. While the limits may be higher, motions are measured at bearing locations. In addition, operators of reciprocating compressors often monitor vibrations of the compressor base relative to the foundation (sometimes called “frame movement”) as a measure of the foundation and machine-mounting condition and integrity.

Impulsive machines (presses, forging hammers) tend not to have specific vibration limitations as controllable by the foundation design. With these machines, it is important to recognize the difference between the inertial forces and equipment dynamics as contrasted with the foundation system dynamics. The forces with the equipment can generate significant accelerations and stresses that are unrelated to the stiffness, mass, or other design aspect of the foundation system. Thus, monitoring accelerations in particular on an equipment frame may not be indicative of foundation suitability or adequacy.

Researchers have presented various studies and papers addressing the issues of machinery vibration limits. This variety is reflected in the standards of engineering companies, plant owners, and industry standards. When the equipment manufacturer does not establish limits, recommendation from ISO 10816-1, Blake (1964), and Baxter and Bernhard (1967) are often followed. Most of these studies relate directly to rotating equipment. In many cases they are also applicable to reciprocating equipment. Rarely do these studies apply to impulsive equipment.

ISO publishes ISO 10816 in a series of six parts to address evaluation of machinery vibration by measurements on the nonrotating parts. Part 1 provides general guidelines and sets the overall rules with the subsequent parts providing specific values for specific machinery types. These standards are primarily directed toward in-place measurements for the assessment of machinery operation. They are not intended to identify design standards. Design engineers, however, have used predecessor documents to ISO 10816 as a baseline for design calculations and can be expected to do similarly with these more recent standards.

The document presents vibration criteria in terms of rms velocity. Where there is complexity in the vibration signal (beyond simple rotor unbalance), the rms velocity basis provides the broad measure of vibration severity and can be correlated to likely machine damage. For situations where the pattern of motion is fairly characterized by one simple harmonic, such as simple rotor unbalance, the rms velocities can be multiplied by √2 to determine corresponding peak velocity criteria. For these same cases, displacements can be calculated as
where

\[ v = \frac{v'}{\omega_m} = \frac{v'}{2\pi f_m} \quad (3-37) \]

Table 3.4—Service factors from Blake (Richart, Hall, and Woods 1970)

<table>
<thead>
<tr>
<th>Item</th>
<th>Bolted down</th>
<th>Not bolted down</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-stage centrifugal pump, electric motor, fan</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Typical chemical processing equipment, noncritical</td>
<td>1.0</td>
<td>0.4</td>
</tr>
<tr>
<td>Turbine, turbo-generator, centrifugal compressor</td>
<td>1.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Centrifugal, stiff-shaft (at basket housing), multi-stage centrifugal pump</td>
<td>2.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Miscellaneous equipment, characteristics unknown</td>
<td>2.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Centrifuge, shaft-suspended, on shaft near basket</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>Centrifuge, link-suspended, slung</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Notes: 1. Vibration is measured at the bearing housing except as noted; 2. Machine tools are excluded; and 3) Compared or measured displacements are multiplied by the appropriate service factor before comparing with Fig. 3.9.

The subsequent parts of ISO 10816 establish the boundaries between these zones as applicable to specific equipment. Part 2, ISO 10816-2, establishes criteria for large, land-based, steam-turbine generator sets rated over 67,000 horsepower (50 MW). The most general of the standards is Part 3, ISO 10816-3, which addresses in-place evaluation of general industrial machinery nominally more than 15 kW and operating between 120 and 15,000 rpm. Within ISO 10816-3, criteria are established for four different groups of machinery, and provisions include either flexible or rigid support conditions. Criteria are also established based on both rms velocity and rms displacement. Part 4, ISO 10816-4, identifies evaluation criteria for gas-turbine-driven power generation units (excluding aircraft derivatives) operating between 3000 and 20,000 rpm. Part 5 (ISO 10816-5) applies to machine sets in hydro-power facilities and pumping plants. Part 6, ISO 10816-6, provides evaluation criteria for reciprocating machines with power ratings over 134 horsepower (100 kW). The scope of Part 5 is not applicable to general equipment foundations and the criteria of Part 6 are not sufficiently substantiated and defined to be currently useful.

Another document available for establishing vibration limitation is from Lifshits (Lifshits, Simmons, and Smalley 1986). This document follows Blake’s approach of identifying five different categories from No Faults to Danger of Immediate Failure. In addition, a series of correction factors are established to broaden the applicability to a wider range of equipment and measurement data.

Blake’s paper (Blake 1964) has become a common basis for some industries and firms. His work presented a standard vibration chart for process equipment with performance rated from “No Faults (typical of new equipment)” to “Dangerous (shut it down now to avoid danger).” The chart was primarily intended to aid plant personnel in assessing field installations and determining maintenance plans. Service factors for different types of equipment are used to allow widespread use of the basic chart. This tool uses vibration displacement (in. or mm) rather than velocity and covers speed ranges from 100 to 10,000 rpm. Figure 3.9 and Table 3.4 present the basic chart and service factors established by Blake.

Baxter and Bernhard (1967) offered more general vibration tolerances in a paper that has also become widely referenced. Again with primary interest to the plant maintenance operations, they established the General Machinery Vibration Severity Chart, shown in Fig. 3.10, with severity ranging from extremely...
smooth to very rough. These are plotted as displacement versus vibration frequency so that the various categories are differentiated along lines of constant peak velocity.

The American Petroleum Institute (API) also has a series of standards for equipment common in the petrochemical industry (541, 610, 612, 613, 617, 618, and 619). ISO 10816-3 can be applied for some large electrical motors; however, most design offices do not generally perform rigorous analyses for these items.

Figure 3.10 provides a comparison of five generic standards against four corporate standards. To the extent possible, the comparisons are presented on a common basis. In particular, the comparison is based on equipment that is in service, perhaps with minor faults, but which could continue in service indefinitely. The Blake line is at the upper limit of the zone identifying operation with minor faults with a service factor of one applicable for fans, some pumps, and similar equipment. The Lifshits line separates the acceptable and marginal zones and includes a $K$ of 0.7, reflecting equipment with rigid rotors. The ISO lines are drawn at the upper level of Zone B, normally considered acceptable for long-term operation. The ISO 10816-3 line is for large machines between 400 and 67,000 horsepower (300 kW and 50 MW) on rigid support systems. The ISO 10816-2 is for large turbines over 67,000 horsepower (50 MW).

The company standards are used for comparison to calculate motions at the design stage. For these calculations, the companies prescribe rotor unbalance conditions worse than those expected during delivery and installation. These load definitions are consistent with those presented in Section 3.2.2.1. Thus, there is a level of commonality. Company G’s criteria are for large turbine applications and, thus, most comparable to the ISO 10816-2 criteria. The other company standards are for general rotating equipment. Company F permits higher motions for reciprocating equipment. In all cases the design companies standards reflect that the manufacturer may establish equipment-specific criteria that could be more limiting than their internal criteria.

Figure 3.11 shows that the corporate standards are generally below the generic standards because the generic standards are intended for in-place service checks and maintenance decisions rather than offering initial design criteria. One company is clearly more lenient for very low-speed equipment, but the corporate standards tend to be similar.

The Shock and Vibration Handbook (Harris 1996) contains further general information on such standards.

3.4.2 Physiological limits—Human perception and sensitivity to vibration is ambiguous and subjective. Researchers have studied and investigated this topic, but there are no clear uniform U.S. standards. In Germany, VDI 2057 provides guidance for the engineer. Important issues are the personnel expectations and needs and the surrounding environment.

ISO 2631 provides guidance for human exposure to whole-body vibration and considers different comfort levels and duration of exposure. This document does not address the extensive complexities identified in ISO 2631. Figure 3.12 presents the basic suggested acceleration limits from ISO 2631 applicable to longitudinal vibrations (vertical for a standing person). This figure reflects the time of exposure and frequency consideration for fatigue-decreased proficiency. The figure shows that people exhibit fatigue and reduced proficiency when subjected to small accelerations for long periods or greater accelerations for shorter periods. The frequency of the accelerations also impact fatigue and proficiency.

The modified Reiher-Meister figure (barely perceptible, noticeable, and troublesome) is also used to establish limits with respect to personnel sensitivity, shown in Fig. 3.13.
DIN 4150 is another standard used internationally. Part 3 defines permissible velocities suitable for assessment of short-term vibrations on structures, which are given in Table 3.5. Furthermore, Part 2 of this German standard defines limitations for allowable vibrations based on perception as a function of location (residential, light industrial) and either daytime or nighttime. Most engineering offices do not consider human perception to vibrations, unless there are extenuating circumstances (proximity to office or residential areas).

There are no conclusive limitations on the effects of vibration of surrounding buildings. The Reiher-Meister figure identifies levels of vibration from mining operations that have damaged structures.

### Frequency ratios

The frequency ratio is a term that relates the operating speed of the equipment to the natural frequencies of the foundation. Engineers or manufacturers require that the frequency of the foundation differ from the operating speed of the equipment by certain margins. This limitation is applied to prevent resonance conditions from developing within the dynamic soil-foundation-equipment. The formulation or presentation of frequency ratios may be based around either \( \frac{f_0}{f_n} \) or \( \frac{f_n}{f_0} \) (operating frequency to natural frequency or its inverse), and engineers or manufacturers should exercise caution to prevent misunderstandings.

A common practice among engineering firms is to compute the natural frequencies of the basic equipment-foundation and compare the values with the dynamic excitation frequency. Many companies require that the natural frequency be 20 to 33% removed from the operating speed. Some firms have used factors as low as 10% or as high as 50%. If the frequencies are well separated, no further evaluation is needed. If there is a potential for resonance, the engineer should either adjust to the foundation size or perform more refined calculations. Refined calculations may include an analysis with a deliberately reduced level of damping. The size and type of equipment play an important role in this decision process.

Frequency ratio is a reasonable design criterion, but one single limiting value does not fit all situations. Where there is greater uncertainty in other design parameters (soil stiffness, for example), more conservatism in the frequency ratio may be appropriate. Similarly, vibration problems can exist even though resonance is not a problem.

### Transmissibility

A common tool for the assessment of vibrations at the design stage is a transmissibility ratio, as shown in Fig. 3.14, which is based on a single degree-of-freedom (SDOF) system with a constant speed excitation force. This ratio identifies the force transmitted through the spring-damper system with the supporting system as compared with the dynamic force generated by the equipment. This ratio should be as low as possible, that is, only transmit 20% of the equipment dynamic force into the supporting system. Low transmissibility implies low vibrations in the surroundings, but this is not an absolute truth.

This transmissibility figure assumes that the damping force is directly and linearly proportional to the velocity of the SDOF. Where the system characteristics are such that the damping force is frequency dependent, the aforementioned represen-
tation is not accurate. When the damping resistance decreases at higher frequencies, the deleterious effect of damping on force transmissibility can be mitigated.

For soil or pile-supported systems, the transmissibility ratio may not be meaningful. In SDOF models of these systems, the spring and damper are provided by the soil and, while the transmissibility of the design may be low, the energy worked through these system components is motion in the surroundings that may not be acceptable.

3.5—Concrete performance criteria

The design of the foundation should withstand all applied loads, both static and dynamic. The foundation should act in unison with the equipment and supporting soil or structure to meet the deflection limits specified by the machinery manufacturer or equipment owner.

The service life of a concrete foundation should meet or exceed the anticipated service life of the equipment installed and resist the cyclic stresses from dynamic loads. Cracking should be minimized to ensure protection of reinforcing steel.

The structural design of all reinforced concrete foundations should be in accordance with ACI 318. The engineer may use allowable stress methods for nonprestressed reinforced concrete.

In foundations thicker than 4 ft (1.2 m), the engineer may use the minimum reinforcing steel suggested in ACI 207.2R.

API and the Construction Industry Institute published API Recommended Practice 686/PIP REIE 686, “Recommended Practice for Machinery Installation and Installation Design.” Chapter 4 of 686/PIP REIE 686 includes design criteria for soil-supported reinforced concrete foundations that supports general and special purpose machinery. The concrete used in the foundation should tolerate its environment during placement, curing, and service. The engineer should consider various exposures such as freezing and thawing, salts of chlorides and sulfates, sulfate soils, acids, carbonation, repeated wetting and drying, oils, and high temperatures.

In addition to conventional concrete, there are many technologies available—such as admixtures, additives, specialty cements, and preblended products—to help improve placement, durability, and performance properties. These additives include water reducers, set-controlling mixtures, shrinkage-compensating admixtures, polymers, silica fumes, fly ash, and fibers.

Many foundations, whether new or repaired, require a fast turnaround to increase production by reducing downtime without compromising durability and required strength. These systems may use a combination of preblended or field-mixed concrete and polymer concrete or grout to reduce downtime to 12 to 72 h, depending on foundation volume and start-up strength requirements.

3.6—Performance criteria for machine-mounting systems

The machine-mounting system (broadly categorized as either an anchorage-type or an isolator-type) attaches the dynamic machine to its foundation. It represents a vital interface between the machine and the foundation; however, it can suffer from insufficient attention to critical detail by the foundation engineer and machinery engineer because it falls between their areas of responsibility. Anchorage-type machine-mounting systems integrate the foundation and the machine into a single structure. Isolator-type machine-mounting systems separate the machine and the foundation into two separate systems that may still dynamically interact with each other. In the processes of design, installation, and operation, the critical role of both types needs advocacy and the assurance that interface issues receive attention. The research and development of information on machine-mounting system technology by the Gas Machinery Research Council (GMRC) during the 1990s reflects the importance that this group attaches to the anchorage-type machine-mounting system. This research produced a series of reports on machine-mounting topics (Pantermuehl and Smalley 1997a,b; Smalley and Pantermuehl 1997; Smalley 1997). These reports, readily retrievable from www.gmrc.org, are essential for those responsible for dynamic machines and their foundations.

Most large machines, in spite of careful design for integrity and function by their manufacturers, can internally absorb no more than a fraction of the forces or thermal growth inherent in their function.

Those responsible for the machine-foundation interface should provide an attachment that transmits the remaining forces for dynamic integrity of the structure yet accommodates anticipated differential thermal expansion between machine and foundation. They should recognize the inherent conflicts in these requirements, the physical processes that can inhibit performance of these functions, and the lifetime constraints (such as limited maintenance and contaminating materials) from which any dynamic machine can suffer as it contributes to profitable, productive plant operation.

A dynamic machine may tend to get hotter and grow more than its foundation (in the horizontal plane). The growth can reach several tenths of an inch (0.1 in. = 2.54 mm); combustion turbine casings grow so much that they have to include deliberately installed flexibility between hotter and cooler elements of their own metallic structure. Most machines—such as compressors, steam turbines, motors, and generators—do not internally relieve their own thermal growth, so the mounting system should allow for thermal growth. Thermal growth can exert millions of pounds of force (1,000,000 lbf = 4500 kN), a level that cannot be effectively restrained.
Heat is transferred between the machine and foundation through convection, radiation, and conduction. While convection and radiation dominate in the regions where an air gap separates the machine base from the foundation, the mounting system provides the primary path for conduction.

Ten critical performance criteria can be identified as generally applicable to isolator and anchorage-type mounting systems:

1. A machine-mounting system should tolerate expected differential thermal growth across the interface. This can occur by combining strength to resist expansion forces and stresses, flexibility to accommodate the deflections, and tolerance for relative sliding across the interface (as the machine grows relative to the foundation);

2. A machine-mounting system should either absorb or transmit, across the mounting interface, those internally generated dynamic forces, resulting from the machine’s operation not absorbed within the machine structure itself. These forces include both vertical and horizontal components. Flexible mounts that deflect rather than restrain the forces become an option only in cases where the machine and any rigidly attached structure have the structural rigidity needed to avoid damaging internal stresses and deflections. Large machinery may not meet this criterion;

3. A nominally rigid mount should transmit dynamic forces with only microlevel elastic deformation and negligible dynamic slippage across the interface. The dynamic forces should include local forces, such as forces from each individual cylinder, which large machines transmit to the foundation because of their flexibility. For reciprocating compressors, this criterion helps ensure that the foundation and machine form an integrated structure;

4. A machine-mounting system should perform its function for a long life—typically 25 years or more. Specifications from the operator should include required life;

5. Any maintenance and inspection required to sustain integrity of the machine-mounting system should have a frequency acceptable to the operator of the machine, for example, once per year. Engineers, installers, and operators of the machine and its foundation should agree to this maintenance requirement because the design integrity relies on the execution of these maintenance functions with this frequency;

6. The bolts that tie the machine to the mounting system, and which form an integral part of the mounting system, should have sufficient stretch and create enough normal force across all interfaces to meet the force transmission and deflection performance stated above;

7. The anchor bolt material strength should tolerate the resultant bolt tensile stresses. The mounts, soleplates, and grout layers compressed by the anchor bolt should tolerate the compressive stresses imposed on them;

8. Any polymeric material (grout or chocks) compressed by the anchor bolts should exhibit a tolerably low amount of creep to maintain bolt stretch over the time period between maintenance actions performed to inspect and tighten anchor bolts. Indeed, the machine mount should perform its function, accounting for expected creep, even if maintenance occurs less frequently;

9. The mounting system should provide a stable platform from which to align the machine. Any deflections of the mounting system that occur should remain sufficiently uniform at different points to preserve acceptable alignment of the machine. The specifications and use of adjustable chock mounts has become increasingly widespread to compensate for loss of alignment resulting from creep and other permanent deformations; and

10. The mounting system should impose tolerable loads, stresses, and deformations on the foundation itself. Appropriate foundation design to make the loads, stresses, and deformations tolerable remains an essential part of this performance criterion. Some of the loads and stresses to consider include:

- Tensile stresses in the concrete at the anchor bolt termination point, which may cause cracks;
- Shear stresses in concrete above anchor bolt termination points, which, if high enough, might result in pullout;
- Interface shear stresses between a grout layer and the concrete resulting from the typically higher expansion of polymer grout than concrete (best accommodated with expansion joints); and
- Hoggling or sagging deformation of the concrete block produced by heat conduction through the mounting system. Air gaps and low conductivity epoxy chocks help minimize such deformation.

Potential conflicts requiring attention and management in these performance criteria include:

- Requirements to accommodate thermal expansion while transmitting dynamic forces; and
- Requirements to provide a large anchor bolt clamping force (so that slippage is controlled during transfer of high lateral loads) while stresses and deflections in bolt, foundation, chocks, and grout remain acceptably low.

Physical processes that can influence the ability of the mount to meet its performance criteria include:

- Creep—Creep of all polymeric materials under compressive load. Creep means time-dependent deflection under load. Deflection increases with time—sometimes doubling or tripling the initial deflection;

- Differential thermal expansion—This can occur when two adjacent components at similar temperatures have different coefficients of thermal expansion, when two adjacent components of similar coefficients have different temperatures or a combination of both. Machine mounts with epoxy materials can experience both types of differential thermal expansion;

- Friction—Friction is limited by a friction coefficient. Friction defines the maximum force parallel to an interface that the interface can resist before sliding for a given normal force between the two interfacing materials;

- Limits on friction—The presence of oil in the interface (typically cutting the dry friction coefficient in half) causes further limits on friction;

- Yield strength—Yield strength of anchor bolts limits the tension available from an anchor bolt and encourages the use of high-strength anchor bolts for all critical applications;
• **Cracks**—Concrete can crack under tensile loads, and these cracks can grow with time; and

• **Oil**—Oil can pool around many machinery installations. Oil aggravates cracks in concrete, particularly under alternating stresses where it induces a hydraulic action. In many cases, oil, its additives, or the ambient materials it transports react with concrete to reduce its strength, particularly in cracks where stresses tend to be high.

Those responsible for machine mounts, as part of a foundation, should consider the aforementioned performance criteria, the conflicts that complicate the process of meeting those criteria, and the physical processes that inhibit the ability of any installation to meet the performance criteria. Other sections of this document address the calculation of loads, stresses, deflections, and the specific limits of strength implicit in different materials. The GMRC reports referred to address all these issues as they pertain to reciprocating compressors.

### 3.7—Method for estimating inertia forces from multicylinder machines

The local horizontal $F_{zi}$ and vertical $F_{yi}$ unbalanced forces for the $i$-th cylinder located in the horizontal plane can be written as

$$
F_{zi} = (m_{rec,i} + m_{rot,i})r_i\omega_o^2 \cos(\omega_o t + \alpha_i) + \frac{m_{rot,i}r_i^2\omega_o^2}{L_i} \cos(\omega_o t + \alpha_i) \tag{3-38}
$$

and

$$
F_{yi} = m_{rec,i}r_i\omega_o^2 \sin(\omega_o t + \alpha_i) \tag{3-39}
$$

where

- $m_{rec,i}$ = reciprocating mass for the $i$-th cylinder;
- $m_{rot,i}$ = rotating mass of the $i$-th cylinder;
- $r_i$ = radius of the crank mechanism of the $i$-th cylinder;
- $L_i$ = length of the connecting rod of the crank mechanism at the $i$-th cylinder;
- $\omega_o$ = circular operating frequency of the machine (rad/s);
- $t$ = time, s; and
- $\alpha_i$ = the phase angle for the crank radius of the $i$-th cylinder, rad.

The primary and secondary force components are as follows:

(primary)

$$
F'_{zi} = (m_{rec,i} + m_{rot,i})r_i\omega_o^2 \cos(\omega_o t + \alpha_i) \tag{3-40}
$$

$$
F'_{yi} = m_{rec,i}r_i\omega_o^2 \sin(\omega_o t + \alpha_i) \tag{3-41}
$$

(secondary)

$$
F''_{zi} = \left[ m_{rot,i}r_i^2\omega_o^2 \cos(\omega_o t + \alpha_i) \right] \tag{3-42}
$$

If the $i$-th cylinder is oriented at angle $\theta_i$ to a global horizontal z-axis, then the primary and secondary force components, with respect to the global axis, can be rewritten as follows:

(primary)

$$
F_{zi}^{GP} = F'_{zi} \cos \theta_i = F'_{yi} \sin \theta_i \tag{3-43}
$$

$$
F_{yi}^{GP} = F'_{zi} \sin \theta_i = F'_{yi} \cos \theta_i \tag{3-44}
$$

(secondary)

$$
F_{zi}^{GS} = F''_{zi} \cos \theta_i \tag{3-45}
$$

$$
F_{yi}^{GS} = F''_{zi} \sin \theta_i \tag{3-46}
$$

The resultant forces due to $n$ cylinders in global coordinates can be calculated as follows

$$
F_{zi}^{GP} = \sum_{i=1}^{n} F_{zi}^{GP} \tag{3-47}
$$

$$
F_{zi}^{GS} = \sum_{i=1}^{n} F_{zi}^{GS} \tag{3-48}
$$

$$
F_{yi}^{GP} = \sum_{i=1}^{n} F_{yi}^{GP} \tag{3-49}
$$

$$
F_{yi}^{GS} = \sum_{i=1}^{n} F_{yi}^{GS} \tag{3-50}
$$

The resultant moments due to $n$ cylinders in global coordinates can be determined as follows

$$
M_{y}^{GP} = \sum_{i=1}^{n} (F_{zi}^{GP} X_i) \tag{3-51}
$$

$$
M_{y}^{GS} = \sum_{i=1}^{n} (F_{zi}^{GS} X_i) \tag{3-52}
$$

$$
M_{z}^{GP} = \sum_{i=1}^{n} (F_{yi}^{GP} X_i) \tag{3-53}
$$

$$
M_{z}^{GS} = \sum_{i=1}^{n} (F_{yi}^{GS} X_i) \tag{3-54}
$$
where $X_i$ = distance along the crankshaft from the reference origin to the $i$-th cylinder.

Equation (3-43) to (3-54) provide instantaneous values of time-varying inertia (shaking) forces and four time varying shaking moments for an $n$ cylinder reciprocating machine. To visualize the time variation of these forces and moments over a revolution of the crankshaft, they can be computed at a series of crank angle values and plotted against crank angle. To obtain maximum values of the primary and secondary forces and moments (and the phase angle at which the maxima occur), they are computed at two orthogonal angles and vectorially combined as shown as follows:

Maximum global horizontal primary force:

$$
(F^G_P)_\text{max} = \left[\left(F^G_{Z0}\right)^2 + \left(F^G_{Z90}\right)^2\right]^{1/2}
$$

at $\tan^{-1}(F^G_{Z90}/F^G_{Z0})$

Maximum global horizontal secondary force:

$$
(F^G_S)_\text{max} = \left[\left(F^G_{Y0}\right)^2 + \left(F^G_{Y90}\right)^2\right]^{1/2}
$$

at $\tan^{-1}(F^G_{Y90}/F^G_{Y0})$

Maximum global vertical primary force:

$$
(F^G_Y)_{\text{max}} = \left[\left(F^G_{Y0}\right)^2 + \left(F^G_{Y90}\right)^2\right]^{1/2}
$$

at $\tan^{-1}(F^G_{Y90}/F^G_{Y0})$

Maximum global vertical secondary force:

$$
(F^G_S)_\text{max} = \left[\left(F^G_{Z0}\right)^2 + \left(F^G_{Z90}\right)^2\right]^{1/2}
$$

at $\tan^{-1}(F^G_{Z90}/F^G_{Z0})$

Maximum global vertical primary moment:

$$
(M^G_Y)_{\text{max}} = \left[\left(M^G_{Y0}\right)^2 + \left(M^G_{Y90}\right)^2\right]^{1/2}
$$

at $\tan^{-1}(M^G_{Y90}/M^G_{Y0})$

Maximum global vertical secondary moment:

$$
(M^G_S)_{\text{max}} = \left[\left(M^G_{Z0}\right)^2 + \left(M^G_{Z90}\right)^2\right]^{1/2}
$$

where subscripts 0, 45, and 90 represent the value of $\omega_1 t$ used to calculate the force values listed in Eq. (3-55) to (3-62).

CHAPTER 4—DESIGN METHODS AND MATERIALS

4.1—Overview of design methods

4.1.1 General considerations—The objectives of the machine foundation design are to assess the dynamic response of the foundation and verify compliance with the required vibration and structural performance criteria. Machine foundation design includes the following steps:

1. Develop a preliminary size for the foundation using rule-of-thumb approaches, past experience, machine manufacturer recommendations, and other available data;

2. Calculate the vibration parameters, such as natural frequency, amplitudes, velocities, and accelerations, for the preliminarily sized foundation;

3. Verify that these calculated parameters do not exceed recommended limits or vibration performance criteria;

4. If necessary, incorporate appropriate modifications in the foundation design to reduce vibration responses to meet the specified vibration performance criteria and cost; and

5. Check the structural integrity of the concrete foundation and machine-mounting system.

Preventive measures to reduce vibrations are less expensive when incorporated in the original design than remedial measures applied after machinery is in operation. The following are some common means that can be used separately or combined to reduce vibrations:

- Selection of the most favorable location for the machinery;
- Adjustment of machine with respect to speed or balance of moving parts;
- Adjustment of the foundation with respect to mass (larger versus smaller) or foundation type (soil supported versus pile);
- Isolation of the machinery from the foundation using special mountings such as springs and flexible mats; and
- Isolation of the foundations by barriers.

4.1.2 Summary of design methods for resisting dynamic loads—Design methods for the foundations supporting dynamic equipment have gradually evolved over time from
an approximate rule-of-thumb procedure to the scientifically
difficult engineering methods. These methods can be identified
as follows:

- Rule-of-thumb;
- Equivalent static loading; and
- Dynamic analysis.

The selection of an appropriate method depends heavily
on machine characteristics, including unbalanced forces,
speed, weight, center of gravity location, and mounting;
importance of the machine; foundation type and size; and
required performance criteria.

Design of the foundation starts with the selection and assessment of the foundation type, size, and location. Usually, foundation type is governed by the soil properties and operational requirements. The machine footprint, weight, and unbalanced forces govern the size of the foundation. The location of a foundation is governed by environmental and operational considerations. Thus, the engineer should consider information from the following three categories before the foundation can be preliminarily sized:

1. Machine characteristics and machine-foundation performance requirements
   - Functions of the machine;
   - Weight of the machine and its moving components;
   - Location of the center of gravity in both vertical and horizontal dimensions;
   - Speed ranges of the machine;
   - Magnitude and direction of the unbalanced forces and moments; and
   - Limits imposed on the foundation with respect to differential displacement.

2. Geotechnical information
   - Allowable soil-bearing capacity;
   - Effect of vibration on the soil, for example, settlement or liquefaction risk;
   - Classification of soil;
   - Modulus of subgrade reaction;
   - Dynamic soil shear modulus; and
   - Dynamic soil-pile interaction parameters (for pile-supported foundations).

3. Environmental conditions
   - Existing vibrating sources such as existing vibration equipment, vehicular traffic, or construction;
   - Human susceptibility to vibration or vibration-sensitive equipment;
   - Flooding or high water table risk; and
   - Seismic risk.

Usually, the preliminary foundation size is established using the rule-of-thumb method, and then the performance criteria, for both machine and foundation, are verified using the equivalent static loading method or dynamic analysis. If the equivalent static loading method or dynamic analysis shows that the foundation is inadequate, the engineer revises the foundation size and repeats the analysis.

4.1.2.1 Rule-of-thumb method—Rule-of-thumb is one of the simplest design methods for machine foundations resisting vibrations. The concept of this method is to provide sufficient mass in the foundation block so that the vibration waves are attenuated and absorbed by the block and soil system.

Most engineers consider the rule-of-thumb procedure satisfactory for a preliminary foundation sizing. Sometimes engineers use this method to design block-type foundations supporting relatively small machinery, up to 5000 lbf (22 kN) in weight and having small unbalanced forces. For reciprocating machinery and sensitive machinery, rule-of-thumb procedures by themselves may not be sufficient.

A long-established rule-of-thumb for machinery on block-type foundations is to make the weight of the foundation block at least three times the weight of a rotating machine and at least five times the weight of a reciprocating machine. For pile-supported foundations, these ratios are sometimes reduced so that the foundation block weight, including pile cap, is at least 2-1/2 times the weight of a rotating machine and at least four times the weight of a reciprocating machine. These ratios are machine weights inclusive of moving and stationary parts as compared with the weight of the concrete foundation block. Additionally, many designers require the foundation to be of such weight that the resultant of lateral and vertical loads falls within the middle third of the foundation base. That is, the net effect of lateral and vertical loads or the eccentricity of the vertical load should not cause uplift.

The engineer should size the shape and thickness of the foundation to provide uniform distribution of vertical dead and live loads to the supporting soil or piles, if practical. The shape of the foundation should fit the supported equipment requirements. Also, the engineer should provide sufficient area for machine maintenance. The shape of the foundation should adequately accommodate the equipment, including maintenance space if required. Minimum width should be 1.5 times the vertical distance from the machine centerline to the bottom of the foundation block. The designer should adjust the length and width of the foundation so the center of gravity of the machine coincides with the center of gravity of the foundation block in plan. A common criteria is that the plan view eccentricities between the center of gravity of the combined machine-foundation system and the center of resistance (center of stiffness) should be less than 5% of the plan dimensions of the foundation. In any case, the foundation is sized so that the foundation bearing pressure does not exceed the allowable soil-bearing capacity.

Thickness criteria primarily serve to support a common assumption that the foundation behaves as a rigid body on the supporting material. Clearly, this is a more complex problem than is addressed by simple rules-of-thumb. On soft materials, a thinner section may be sufficient, whereas on stiffer soils, a thicker section might be required to support the rigid body assumption. If the rigid body assumption is not applicable, more elaborate computation techniques, such as finite element methods, are used. Gazetas (1983) provides some direction in this regard. One rule-of-thumb criterion for thickness is that the minimum thickness of the foundation block should be 1/5 of its width (short side), 1/10 of its length (long side), or 2 ft (0.6 m), whichever is greatest. Another criterion is given in Section 4.3 as 1/30 of the length plus 2 ft (0.6 m).
The designer may need to provide isolation or separation of the machine foundation from the building foundation or slab. Separation in the vertical direction may also be appropriate. Normally, dynamically loaded foundations are not placed above building footings or in such locations that the dynamic effects can transfer into the building footings.

4.1.2.2 Equivalent static loading method—The equivalent static loading method is a simplified and approximate way of applying pseudodynamic forces to the machine-supporting structure to check the strength and stability of the foundation. This method is used mainly for the design of foundations for machines weighing 10,000 lbf (45 kN) or less.

For design of reciprocating machine foundations by the static method, the machine manufacturer should provide the following data:

- Weight of the machine;
- Unbalanced forces and moments of the machine during operation; and
- Individual cylinder forces including fluid and inertia effects.

For design of rotating machine foundations by the static method, the machine manufacturer should provide:

- Weight of the machine and base plate;
- Vertical pseudodynamic design force; and
- Horizontal pseudodynamic design forces—lateral force and longitudinal force.

Calculated natural frequencies, deformations, and forces within the structure supporting the machine should satisfy established design requirements and performance criteria outlined in Section 3.4.

4.1.2.3 Dynamic analysis—Dynamic analysis incorporates more advanced and more accurate methods of determining vibration parameters and, therefore, is often used in the final design stage and for critical machine foundations. Dynamic analysis (or vibration analysis) is almost always required for large machine foundations with significant dynamic forces (Section 4.3).

For the dynamic analysis of reciprocating and rotating machine foundations, the machine manufacturer should provide the following data to the extent required by the analysis:

- Primary unbalanced forces and moments applied at the machine speed over the full range of specified operating speeds;
- Secondary unbalanced forces and moments applied at twice the machine speed over the full range of specified operating speeds; and
- Individual cylinder forces including fluid and inertia effects.

For the rotating machinery, the dynamic design (Section 4.3) determines the vibration amplitudes based on these dynamic forces. Refer to Section 3.2 for the dynamic force calculations for both reciprocating and rotating machines. When there is more than one rotor, however, amplitudes are often computed with the rotor forces assumed to be in-phase and 180 degrees out-of-phase. To obtain the maximum translational and maximum torsional amplitudes, other phase relationships may also be investigated.

A complete dynamic analysis of a system is normally performed in two stages:

1. Determination of the natural frequencies and mode shapes of a machine-foundation system; and
2. Calculation of the machine-foundation system response caused by the dynamic forces.

Determining the natural frequencies and mode shapes of the machine-foundation system provides information about the dynamic characteristic of that system. In addition, calculating the natural frequencies identifies the fundamental frequency, usually the lowest value of the natural frequencies.

The natural frequencies are significant for the following reasons:

- They indicate the relative degree of stiffness of the machine-foundation-soil system; and
- They can be compared with the frequency of the acting dynamic force so that a possible resonance condition may be prevented. Resonance is prevented when the ratio of the machine operating frequency to the fundamental frequency of the machine-foundation system falls outside the undesirable range (Section 3.4.3). In many cases, six frequencies are calculated consistent with six rigid body motions of the overall machine-foundation system. Any or all of the six frequencies may be compared with the excitation frequency when checking for resonance conditions.

The significance of the mode shapes is as follows:

- They indicate the deflection pattern that the machine-foundation system assumes when it is left to vibrate after the termination of the disturbing force. Generally, it is the first mode that dominates the vibrating shape, and the higher modes supplement that shape when superimposed;
- They indicate the relative degree of the structural stiffness among various points of the machine-foundation system, that is, the relationship between different amplitudes and mode shapes. If the flexural characteristics of a foundation are being modeled, a mode shape may indicate that a portion of the foundation (a beam or a pedestal) is relatively flexible at a particular frequency. For a system represented by a six degree-of-freedom model, the relative importance of the rocking stiffness and the translational stiffness can be indicated;
- They can be used as indicators of sensitivity when varying the stiffness, mass, and damping resistance of the machine-foundation system to reduce the vibration amplitudes at critical points. A particular beam or foundation component may or may not be significant to the overall behavior of the system. An understanding of the mode shapes can help in selecting the most effective approach to alleviating vibration problems.
shape can aid in identifying the sensitive components.

Calculation of the machine-foundation system response caused by the dynamic forces provides the vibration parameters—such as displacement, velocity, and acceleration of the masses—and also the internal forces in the members of the machine support system. Then, these vibration parameters are compared with the defined criteria or recommended allowable values for a specific condition (Sections 4.3 and 3.4), and internal forces are used to check the structural strength of the foundation components.

### 4.2—Impedance provided by the supporting media

The dynamic response of foundations depends on stiffness and damping characteristics of the machine-foundation-soil system. This section presents a general introduction to this subject and a summary of approaches and formulae often used to evaluate the stiffness and damping of both soil-supported and pile foundations. These stiffness and damping relationships, collectively known as impedance, are used for determining both free-vibration performance and motions of the foundation system due to the dynamic loading associated with the machine operation.

The simplest mathematical model used in dynamic analysis of machine-foundation systems is a single degree-of-freedom representation of a rigid mass vertically supported on a single spring and damper combination (Fig. 4.1(a)). This model is applicable if the center of gravity of the machine-foundation system is directly over the center of resistance and the resultant of the dynamic forces (acting through a center of force, CF) is a vertical force passing through center of gravity. The vertical impedance \( k_v^* \) of the supporting media is necessary for this model.

The next level of complexity is a two degree-of-freedom model commonly used when lateral dynamic forces act on a machine-foundation system (Fig. 4.1(b)). Because the lines of action of the forces and the resistance do not coincide, the rocking and translational motions of the system are coupled. For this model, the engineer needs to calculate the horizontal translational impedance \( k_u^* \) and the rocking impedance \( k_\psi^* \) of the supporting media. These impedance values, especially the rocking terms, are usually different in different horizontal directions.

Application of the lateral dynamic forces can cause a machine-foundation system to twist about a vertical axis. To model this behavior, the engineer needs to determine the torsional impedance \( k_\eta^* \) of the supporting media. As in the vertical model, if the in-plan eccentricities between the center of gravity and center of resistance are small, this analysis is addressed with a single degree-of-freedom representation.

When the machine-foundation system is rigid and on the surface, the impedance \( (k_{u1}^*, k_{u2}^*, k_v^*, k_{\psi1}^*, k_{\psi2}^*, k_\eta^*) \) can be represented as six elastic springs (three translational and three angular) and six viscous or hysteretic dampers (three translational and three angular), which act at the centroid of the contact area between the foundation and the supporting media (Fig. 4.1(c)). If the support is provided by piles or isolators, the equivalent springs and dampers act at the center of stiffness of those elements. When the foundation is embedded, additional terms can be introduced as spring and damper elements acting on the sides of the foundation. The effective location of the embedment impedance is determined rationally based on the character of the embedment (Beredugo and Novak 1972; Novak and Beredugo 1972; Novak and Sachs 1973; Novak and Sheta 1980; Lakshmanan and Minai 1981).

The next level of complexity addresses flexible foundations. The structural representation of these foundations is typically modeled using finite element techniques. The flexibility of the structure might be represented with beam-type elements, two-dimensional plane stress elements, plate bending elements, three-dimensional elements, or some combination of these.

The supporting media for these flexible foundations is addressed in one of three basic manners. The most simplistic approach uses a series of simple, elastic, translational springs at each modeled contact point between the structure and the soil. These spring parameters are computed from the spring constants determined assuming a rigid foundation assumption or by some similar rational procedure. The other two approaches are finite element modeling and modeling with boundary elements. Such approaches are not commonly used in day-to-day engineering practice but can be used successfully for major, complex situations where a high degree of refinement is justified.

The basic mathematical model used in the dynamic analysis of various machine-foundation systems is a lumped mass with a spring and dashpot (Fig. 4.2). Lumped mass includes the machine mass, foundation mass, and, in some models, soil mass. A mass \( m \), free to move only in one direction, for

![Fig. 4.1—Common dynamic models.](image-url)
example vertical, is said to have only one degree of freedom (Fig. 4.1(a)). The behavior of the mass depends on the nature of both the spring and the dashpot. The spring, presumed to be massless, represents the elasticity of the system and is characterized by the stiffness constant \( k \). The stiffness constant is defined as the force that produces a unit displacement of the mass. For a general displacement \( v \) of the mass, the force in the spring (the restoring force) is \( kv \).

In dynamics, the displacement varies with time \( t \), thus, \( v = v(t) \). Because the spring is massless, the dynamic stiffness constant \( k \) is equal to the static constant \( k_{st} \), and \( k \) and the restoring force \( kv \) are independent of the rate or frequency at which the displacement varies.

The same concept of stiffness can be applied to a harmonically vibrating column that has its mass distributed along its length (Fig. 4.3(a)). For an approximate analysis at low frequency, the distributed column mass can be replaced by a concentrated (lumped) mass \( m_s \), attached to the top of the column, and the column itself can be considered massless (Fig. 4.3(b)). Consequently, a static stiffness constant \( k_{st} \), independent of frequency, can be used to describe the stiffness of this massless column. The elastic force in the column just below the mass is \( k_{st}v \) for any displacement \( v \). Nevertheless, the total restoring force generated by the column at the top of the lumped mass is the sum of the elastic force in the column and the inertia force of the mass. If the displacement of this mass varies harmonically

\[
v(t) = v \cdot \cos \omega_m t
\]

where
\[\begin{align*}
v & = \text{displacement amplitude; and} \\
\omega_m & = \text{circular frequency of motion.}
\end{align*}\]

Then, the acceleration is

\[
\ddot{v} = \frac{d^2 v}{dt^2} = -v \cdot \omega_m^2 \cos \omega_m t
\]

and the inertia force is

\[
m_s \cdot \ddot{v} = -m_s \cdot v \cdot \omega_m^2 \cos \omega_m t
\]

where \( m_s \) is the effective mass of a spring.

In the absence of damping, the time-dependent relation of the external harmonic force applied with amplitude \( F \) and frequency \( \omega_o \) to the displacement \( v \) is

\[
F \cdot \cos \omega_o t = k_{st} \cdot v \cdot \cos \omega_o t - m_s \cdot v \cdot \omega_m^2 \cos \omega_m t
\]

and the amplitudes of force and displacement relate as

\[
F = k_{st} \cdot v - m_s \cdot v \cdot \omega_o^2
\]

The dynamic stiffness, being the constant of proportionality between the applied force and displacement, becomes

\[
k = k_{st} - m_s \omega_o^2
\]

Thus, with vibration of an element having distributed mass, the dynamic stiffness generally varies with frequency. At low frequency, this variation is sometimes close to parabolic, as shown on Fig. 4.3(c). The column used in this example may be the column of soil and, thus, a soil deposit may feature stiffness parameters that are frequency dependent. The magnitude and character of this frequency effect depend on the size of the body, vibration mode, soil layering, and other factors.

The dashpots of Fig. 4.2 are often represented as viscous dampers producing forces proportional to the vibration velocity \( \dot{v} \) of the mass. The magnitude of the damper force is

\[
F_D = c \dot{v} = c \frac{dv}{dt}
\]

where
\[\begin{align*}
F_D & = \text{damper force, lbf (N); and} \\
c & = \text{viscous damping constant, lbf-s/ft (N-s/m).}
\end{align*}\]
The damping constant \( c \) is defined as the force associated with a unit velocity. During harmonic vibrations, the time-dependent viscous damping force is

\[
cv = -c v \omega_m \sin \omega_m t
\]  

(4-8)

and its peak value is \( c v \omega_m \). Equation (4-8) shows that for a given constant \( c \) and displacement amplitude \( v \), the amplitude of viscous damping force is proportional to frequency (Fig. 3.8).

Several ways are used to account for variations of the dynamic stiffness with frequency in the soil.

- **Frequency-dependent equations**—Veletsos and others (Veletsos and Nair 1974; Veletsos and Verbic 1973; and Veletsos and Wei 1971) have developed appropriate equations that represent the impedance to motion offered by uniform soil conditions. They developed these formulations using an assumption of uniform elastic or viscoelastic half-space and the related motion of a rigid or flexible foundation on this half-space. Motions can be translational or rotational. While the calculations can be manually tedious, computer implementation provides acceptable efficiency. Some of the simpler relevant equations are presented in later sections of this report. More extensive equations are presented in the references; equations are presented in the references.

- **Constant approximation**—If the frequency range of interest is not very wide, an often satisfactory technique is to replace the variable dynamic stiffness by the constant representative of the stiffness in the vicinity of the dominant frequency (Fig. 4.3(c)). Another constant approximation can be to add an effective or fictitious in-phase mass of the supporting soil medium to the vibrating lumped mass of the machine and foundation and to consider stiffness to be constant and equal to the static stiffness. Nevertheless, the variation of dynamic stiffness with frequency can be represented by a parabola only in some cases and the added mass is not the same for all vibration modes; and

- **Computer-based numerical analysis**—For complex geometry or variable soil conditions, the trend is to obtain the stiffness and damping of foundations using computer programs. The real domain solution is more easily understood on a subjective basis and is typified by the damped stiffness models of Richart and Whitman (Richart and Whitman 1967; Richart, Hall, and Woods 1970), in which the stiffness and damping are represented as constants. The complex domain impedance is easier to describe mathematically and is applied in the impedance models of Veletsos and others (Veletsos and Nair 1974; Veletsos and Verbic 1973; Veletsos and Wei 1971). Equation (4-10) characterizes the relationship between impedance models and damped stiffness models.

\[
k_i^* = k_i + i \omega_m c_i
\]

(4-10)

where
Another common approach is to calculate impedance parameters based on a dimensionless frequency $a_\omega$ computed as

$$a_\omega = R \cdot \omega_m / V_s = R \cdot \omega_m \cdot \sqrt{\rho / G}$$

where

- $R$ = equivalent foundation radius (Eq. (4.9a), (4.9b), or (4.9c));
- $\omega_m$ = circular frequency of motion;
- $V_s$ = shear wave velocity of the soil;
- $\rho$ = soil mass density; and
- $G$ = dynamic shear modulus of the soil.

In the most common circumstances, the soil and foundation are moving at the same frequency the machine operates at. Thus, $a_\omega$ is commonly equal to $a_\omega'$. In unusual cases such as when motions result from a multiple harmonic excitation of the equipment speed, from blade passage effects, from multiple speed equipment combinations, and from some poor bearing conditions, the motion may develop at a speed different from the equipment speed. These situations are more commonly found during in-place problem investigations rather than initial design applications. In those cases, the correct frequency to consider is determined by in-place field measurements.

**4.2.1 Uniform soil conditions**—In many instances, engineers approximate or assume that the soil conditions are uniform throughout the range of interest for the foundation being designed. This assumption in some cases is reasonable. In other cases, it is accepted for lack of any better model and because the scope of the foundation design does not warrant more sophisticated techniques. These are commonly referred to as half-space models.

**4.2.1.1 Richart-Whitman models**—Engineers frequently use a lumped parameter model (Richart and Whitman 1967), which is considered suitable for uniform soil conditions. This model represents the stiffness, damping, and mass for each mode as singular, lumped parameters. In the norm, these parameters are treated as frequency-independent. For the vertical direction, the stiffness and damping equations are expressly validated for the range of dimensionless frequencies from 0 to 1.0 (0 < $a_\omega < 1.0$) (Richart, Hall, and Woods 1970). Implicitly, the other directions are similarly limited, as the stiffness parameters are actually static stiffness values. Often, presentations of the data extend the range to $a_\omega$ values as high as 1.5 to 2.0.

The mass of these models is determined solely as the translational mass and rotational mass moment of inertia for the appropriate directions. No effective soil mass is included in the representation (Richart, Hall, and Woods 1970).

The damping recommended by Richart (Richart, Hall, and Woods 1970) associates with motion of rigid, circular foundations as given in the following equations:

**Vertical:** $k_v = \frac{4}{(1 - \nu)} GR$  \hspace{1cm} (4-13a)

**Horizontal:** $k_h = \frac{32(1 - \nu)}{(7 - 8\nu)} GR$  \hspace{1cm} (4-13b)

where

- $B_i$ = mass ratio for the $i$-th direction;
- $D_i$ = damping ratio for the $i$-th direction;
- $m$ = mass of the machine-foundation system;
- $I_i$ = mass moment of inertia of the machine-foundation system for the $i$-th direction;
- $\rho$ = soil mass density; and
- $R, R_i$ = effective radius of the foundation for the applicable direction.

The lumped parameter models generally recognize two alternate representations for the lumped stiffness terms. These equations are based on the theory of elasticity for elastic half-spaces with rigid foundations, except for horizontal motions, which make a slightly different rigidity assumption. For a rigid circular foundation the applicable formulas are given in Eq. (4-13a) to (4-13d). For a rectangular foundation, the equivalent radii of Eq. (4-9) can be used in the calculations. Alternatively, Eq. (4-13e) to (4-13g) can be used directly for rectangular foundations of plan dimensions $a$ by $b$. In these equations, the $B_i$ values vary with the aspect ratio of the rectangle. The $\beta_\nu$ value for the vertical direction ranges from approximately 2.1 to 2.8, $\beta_u$ for the horizontal ranges from approximately 0.95 to 1.2, and $\beta_\psi$ for the rocking varies from approximately 0.35 to 1.25. Specific relationships can be found in a variety of sources (Richart, Hall, and Wood 1970; Ayra, O’Neill, and Pincus 1979; Richart and Whitman 1967), usually in a graphical form. The original source materials contain the mathematical relationships. There is little difference between using either the circular or the rectangular formulations for these stiffness terms.

**Vertical:** $B_v = \frac{(1 - \nu)}{4} \frac{m}{\rho R^3}$ and $D_v = \frac{0.425}{\sqrt{B_v}}$  \hspace{1cm} (4-12a)

**Horizontal:** $B_u = \frac{(7 - 8\nu)}{32(1 - \nu)} \frac{m}{\rho R^3}$ and $D_u = \frac{0.288}{\sqrt{B_u}}$  \hspace{1cm} (4-12b)

**Rocking:** $B_\psi = \frac{3(1 - \nu)}{8} \frac{I_u}{\rho R^3}$ and $D_\psi = \frac{0.50}{(1 + 2B_\psi)}$  \hspace{1cm} (4-12c)

**Torsion:** $B_\eta = \frac{I_\eta}{\rho R^3}$ and $D_\eta = \frac{0.50}{(1 + 2B_\eta)}$  \hspace{1cm} (4-12d)
Rocking: \( k_\psi = \frac{8}{3(1 - \nu)} G R^3_\psi \) \hspace{1cm} (4-13c)

\[ \chi_\psi = \frac{\beta_1 (\beta_2 a_o)^2}{1 + (\beta_2 a_o)^2} \]

Torsion: \( k_\eta = \frac{16}{3} G R^3_\eta \) \hspace{1cm} (4-13d)

\[ \Psi_\psi = \frac{\beta_1 \beta_2 \cdot (\beta_2 a_o)^2}{1 + (\beta_2 a_o)^2} \]

Vertical: \( k_v = \frac{G}{(1 - \nu)} \beta_v \sqrt{ab} \) \hspace{1cm} (4-13e)

Vertical impedance

\[ k_v^* = \frac{4GR}{(1 - \nu)} [(1 - \chi_v - \gamma_2 a_o^2) + i a_o (\gamma_4 + \psi_v)] \] \hspace{1cm} (4-15c)

where

- \( k_i \) = stiffness for the \( i \)-th direction;
- \( G \) = shear modulus of the soil;
- \( R, R_i \) = equivalent foundation radius;
- \( \nu \) = Poisson’s ratio of the soil;
- \( a, b \) = plan dimensions of a rectangular foundation; and
- \( \beta_i \) = rectangular footing coefficients (Richart, Hall, and Woods 1970).

The damping constants can be determined from the computed damping ratio, system mass, and stiffness as

\[ c_i = 2D_i \sqrt{k_i m} \text{ or } c_i = 2D_i \sqrt{k_i l_i} \] \hspace{1cm} (4-14)

where

- \( c_i \) = damping constant for the \( i \)-th direction;
- \( D_i \) = damping ratio for the \( i \)-th direction;
- \( k_i \) = stiffness for the \( i \)-th direction;
- \( m \) = mass of the machine-foundation system; and
- \( l_i \) = mass moment of inertia of the machine-foundation system for the \( i \)-th direction.

**4.2.1.2 Veletsos models**—For the impedance functions of foundations resting on the surface of a viscoelastic half-space, Veletsos and Verbic (1973) determined the analytical expressions for impedance dependent on frequency, Poisson’s ratio, and internal material damping. Neglecting the internal material damping, the relationships are

Horizontal impedance:

\[ k_\psi^* = \frac{8GR}{2 - \nu} [1 + ia_o \alpha_\eta] \] \hspace{1cm} (4-15a)

Rocking impedance

\[ k_\psi^* = \frac{8GR^3_\psi}{3(1 - \nu)} [(1 - \chi_\psi - \beta_2 a_o^2) + ia_o \psi_\psi] \] \hspace{1cm} (4-15b)

where

- \( k_i^* \) = impedance in the \( i \)-th direction;
- \( R, R_i \) = equivalent foundation radius;
- \( G \) = dynamic shear modulus of the soil;
- \( a_o \) = dimensionless frequency;
- \( \nu \) = Poisson’s ratio of the soil;
- \( \alpha_\eta, \beta_j, \text{ and } \gamma_j \) = coefficients dependent on Poisson’s ratio as given in Table 4.11.
- \( j \) = 1 to 4 as appropriate; and
- \( b_1, b_2 \) = 0.425 and 0.687, respectively (Veletsos and Nair 1974).

The leading terms of each of these impedance equations are the static stiffnesses of the foundation for motion in that direction. For three cases (vertical, rocking, and torsional motion), these terms are the same as presented in the Richart-Whitman lumped parameter model. For the horizontal motion, there is a slight difference due to assumptions about the foundation rigidity. The practicality of the difference is small, especially toward the higher range of Poisson’s ratio values.

These equations may also be expressed as
Table 4.1—Values of $\alpha_i$, $\beta_i$, and $\gamma_j$

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$\nu = 0$</th>
<th>$\nu = 0.33$</th>
<th>$\nu = 0.45$</th>
<th>$\nu = 0.50$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.775</td>
<td>0.650</td>
<td>0.600</td>
<td>0.600</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.525</td>
<td>0.500</td>
<td>0.450</td>
<td>0.400</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.800</td>
<td>0.800</td>
<td>0.800</td>
<td>0.800</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.023</td>
<td>0.027</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.250</td>
<td>0.350</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>1.000</td>
<td>0.800</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>0.000</td>
<td>0.000</td>
<td>—</td>
<td>0.170</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>0.850</td>
<td>0.750</td>
<td>—</td>
<td>0.850</td>
</tr>
</tbody>
</table>

Table 4.2—Stiffness and damping parameters ($D = 0$)

<table>
<thead>
<tr>
<th>Motion</th>
<th>Soil</th>
<th>Side layer</th>
<th>Half space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>Cohesive</td>
<td>$S_{u1} = 4.1$</td>
<td>$S_{u2} = 10.6$</td>
</tr>
<tr>
<td></td>
<td>Granular</td>
<td>$S_{u1} = 4.0$</td>
<td>$S_{u2} = 9.1$</td>
</tr>
<tr>
<td>Rocking</td>
<td>Cohesive</td>
<td>$S_{v1} = 2.5$</td>
<td>$S_{v2} = 1.8$</td>
</tr>
<tr>
<td></td>
<td>Granular</td>
<td>$S_{v1} = 2.5$</td>
<td>$S_{v2} = 1.8$</td>
</tr>
<tr>
<td>Torsion</td>
<td>Cohesive</td>
<td>$S_{c1} = 10.2$</td>
<td>$S_{c2} = 5.4$</td>
</tr>
<tr>
<td></td>
<td>Granular</td>
<td>$S_{c1} = 10.2$</td>
<td>$S_{c2} = 5.4$</td>
</tr>
<tr>
<td>Vertical</td>
<td>Cohesive</td>
<td>$S_{u1} = 2.7$</td>
<td>$S_{u2} = 6.7$</td>
</tr>
<tr>
<td></td>
<td>Granular</td>
<td>$S_{u1} = 2.7$</td>
<td>$S_{u2} = 6.7$</td>
</tr>
</tbody>
</table>

Note: $S$ values are valid for $0.5 < \alpha < 1.5$; $C$ values for valid for $\alpha < 2.0$.

Horizontal impedance: $k_{i\nu} = GR(C_{u1} + ia_oC_{u2})$ (4-16a)

Rocking impedance: $k_{i\nu} = GR^3(C_{v1} + ia_oC_{v2})$ (4-16b)

Vertical impedance: $k_{i\nu} = GR(C_{c1} + ia_oC_{c2})$ (4-16c)

Rocking impedance: $k_{i\nu} = GR^3(C_{c1} + ia_oC_{c2})$ (4-16d)

where

- $k_{i\nu}$ = impedance in the $i$-th direction;
- $R, R_i$ = equivalent foundation radius;
- $G$ = dynamic shear modulus of the soil; and
- $C_{11}, C_{12}$ = dimensionless parameters.

Constant approximations of $C_{11}$ and $C_{12}$ are given in Table 4.2 for two broad classes of soils (cohesive and granular) and for dimensionless frequency values $a_o$ less than 2.0. Polynomial expansions of these approximations are also available to cover a wider range of dimensionless frequencies.

4.2.1.3 Other models—Engineers have used finite element models as another approach to obtain stiffness and damping relationships. Rather than theoretically assessing the behavior of an elastic continuum, parametric studies with a finite element model determine the impedance relationships. Given all the approximations involved, the agreement between the solutions—the half-space theory and the finite element modeling—is very good. The exception is the rocking stiffness $k_{vr}$ for which researchers have computed substantially larger values by finite element solutions (Karabalis and Beskos 1985; Kobayashi and Nishimura 1983; Wolf and Darbre 1984). The empirical expressions for the static stiffness of circular foundations embedded into a homogeneous soil layer can be found in Elsabee and Murray (1977) and Kausel and Ushijima (1979).

4.2.2 Adjustments to theoretical values—Damping in a soil-foundation system consists of two components: geometric damping and material damping. The geometric damping, also called radiation damping, represents a measure of the energy radiated away from the immediate region of the foundation by the soil. Material damping measures the energy loss as a result of soil hysteresis effects.

Sections 4.2.1.1 and 4.2.1.2 present equations to evaluate geometric damping that can be evaluated for various shape foundations and different types of soils using elastic half-space theories. Both experience and experimental results show that damping values for large foundations undergoing small vibration amplitudes are typically less than those analytically predicted values (EPRI 1980; Novak 1970). This discrepancy is attributed to the presence of soil layers that reflect waves back to the vibrating foundation. Various sources recommend reduced soil values (EPRI 1980; Novak 1970; DIN 4024; and Gazetas 1982). Specific recommendations vary with the type of application. The foundation engineer needs to select proper soil damping values and limits based on the specific application. For example, EPRI 1980 recommends the soil damping ratio for use in the design of power plant fan foundations should not exceed 20% for horizontal motion, 50% for vertical motion, 10% for transverse rocking motion, and 15% for axial and torsional motions. German DIN 4024 recommends that the soil damping ratios used in vibration analysis of rigid block foundations should not exceed 25%. Novak (1970) recommends reducing the analytically determined geometric damping ratios (from elastic half-space models) by 50% for a dynamic analysis of the foundation.

Although material damping is often neglected, as presented in Section 4.2.1, that assumption often leads to overestimating the first resonant amplitude of the coupled translation/rocking response of surface footings by several hundred percent because very limited geometric damping develops during rocking. This overestimation can be reduced by the inclusion of material damping (Section 4.2.4).

Torsional response is difficult to predict because of slippage. For surface foundations, slippage reduces stiffness and increases damping; for embedded foundations, slippage reduces damping. The inclusion of the weakened zone around the footing may improve the agreement between the theory and the experiments. This typically requires more complete computer-based numerical analysis.

Another correction of the half-space theory may be required if the soil deposit is a shallow layer. In such cases, the stiffness increases and geometric damping decreases and can even vanish if the frequency of interest (for example, the excitation frequency) is lower than the first natural frequency of the soil layer (Kausel and Ushijima 1979). For a homogeneous layer of thickness $H$ with soil shear wave velocity $V_S$, the first natural frequencies of the soil deposit are...
Horizontal direction: \( \omega_{su} = \frac{\pi V_s}{2H} \frac{2(1 - \nu)}{1 - 2\nu} \)  
Vertical direction: \( \omega_{sv} = \frac{\pi V_s}{2H} \)  

where \( \omega_{su}, \omega_{sv} = \) circular natural frequencies of a soil layer in \( u \) and \( v \) directions;  
\( V_s = \) soil shear wave velocity;  
\( H = \) depth of soil layer; and  
\( \nu = \) Poisson’s ratio of the soil.

At excitation frequencies \( \omega \) lower than \( \omega_{su} \) and \( \omega_{sv} \), only material damping remains because no progressive wave occurs to generate geometric damping in the absence of material damping, and only a very weak progressive wave occurs in the presence of material damping. The damping parameters generated by the material alone for use at excitation frequencies \( \omega \) lower than \( \omega_{su} \) and \( \omega_{sv} \) are

\[
\begin{align*}
c_u &= 2\beta_m \cdot k_u / \omega_o \\
c_v &= 2\beta_m \cdot k_v / \omega_o
\end{align*}
\]

where \( \beta_m = \) material damping ratio, and other terms are as previously defined. In the impedance models, the imaginary terms of the horizontal \( (k_u^*) \) and vertical \( (k_v^*) \) impedance become \( +2\beta_m \omega \) within the brackets of Eq. (4-15a) and (4-15c). This correction is most important for the vertical and horizontal directions, in which the geometric damping of the half-space is high but is minimal for the shallow layer. This correction is not an absolute breakpoint based on the computed layer frequency and the excitation frequency. Short of using computer-based numerical solution techniques that reasonably represent the loss of geometric damping, the foundation engineer should apply judgement to decrease the geometric damping for shallow layer sites.

4.2.3 Embedment effects—Most footings do not rest on the surface of the soil but are partly embedded. Studies have shown that embedment increases both stiffness and damping, but the increase in damping is more significant.

Overall, embedment effects are often overestimated because soil stiffness (shear modulus) diminishes toward the soil surface due to diminishing confining pressure. This is particularly so for backfill lacking a stiff surface crust and whose effects are always much less pronounced than those of undisturbed soil. The lack of confining pressure at the surface often leads to separation of the soil from the foundation and to the creation of a gap as indicated on Fig. 4.5, which significantly reduces the effectiveness of embedment. To find an approximate correction for this effect, the engineer should consider an effective embedment depth less than the true embedment.

In determining the stiffness of embedded foundations, an approximate but versatile approach was formulated using the assumption that the soil reactions acting on the base can be taken as equal to those of a surface foundation (half-space) and the reactions acting on the footing sides as equal to those of an independent layer overlying the half-space (Fig. 4.5). The evaluation of the reactions of a layer is simplified and calculated using the assumption of plane strain. This means that these reactions are taken as equal to those of a rigid, infinitely long, massless cylinder undergoing a uniform motion in an infinite homogeneous medium.

The plane strain approach to the side reactions has many advantages: it accounts for energy radiation through wave propagation, leads to closed-form solutions, and allows for the variation of the soil properties with depth. It can also allow for the slippage zone around the footing. Also, the approach is simple and makes it possible to use the solutions of surface footings because the effect of the independent side layer actually represents an approximate correction of the half-space solutions for the embedment effect. This approach works quite well, and its accuracy increases with increasing frequency.

Equation (4-21a) to (4-21d) describe the side resistance of the embedded cylinder, analogous to the surface disc, using complex, frequency-dependent impedance

Horizontal impedance: \( k_{eu}^* = G_s l[S_{u1} + i \cdot a_o \cdot S_{u2}] \)  
Vertical impedance: \( k_{ev}^* = G_s l[S_{v1} + i \cdot a_o \cdot S_{v2}] \)  
Rocking impedance:  
\[
k_{e\psi}^* = G_s R_e^2 [S_{\psi1} + i \cdot a_o \cdot S_{\psi2}]
\]
Torsional impedance:  
\[
k_{e\eta}^* = G_s R_e^2 [S_{\eta1} + i \cdot a_o \cdot S_{\eta2}]
\]

where  
\( k_{ei}^* = \) impedance in the \( i \)-th direction due to embedment;  
\( G_s = \) dynamic shear modulus of the embedment (side) material;
\[ l = \text{depth of embedment (effective)}; \]
\[ R_i = \text{equivalent foundation radius (Eq. (4.9b) or (4.9c)); and} \]
\[ S_{11}, S_{12} = \text{dimensionless parameters (Table 4.2).} \]

In these expressions, the shear modulus \( G_t \) is that of the side layer that may represent the backfill. The dimensionless parameters \( S_{11} \) and \( S_{12} \) relate to the real stiffness and the damping (out-of-phase component of the impedance), respectively. These parameters depend on the dimensionless frequency \( a_o \) (Eq. (4.11)) applicable for the layer of embedment material. Poisson’s ratio affects only the horizontal impedance generated by the footing embedment, not the impedance in other directions. In complete form, these \( S_i \) parameters also depend on material damping of the side layer soils.

The mathematical expressions for the parameters \( S_{11} \) and \( S_{12} \) can be found in Novak, Nagami, and Aboul-Ella (1978) and Novak and Sheta (1980). These parameters are frequency dependent; nevertheless, given all the approximations involved in the modeling dynamic soil behavior, it is often sufficient to select suitable constant values to represent the parameters, at least over limited frequency range of interest. Table 4.2 shows constant values for cohesive soils and granular soils with Poisson’s ratio of 0.4 and 0.25, respectively. The values correspond to dimensionless frequencies between 0.5 and 1.5, which are typical of many machine foundations. If a large frequency range is important, parameter \( S \) should be considered as frequency dependent and calculated from the expression of impedance functions given in Novak, Nagami, and Aboul-Ella (1978) and Novak and Sheta (1980). Material damping is not included in Table 4.2 but can be accounted for by using Eq. (4.23).

The engineer can approximate the complex stiffness of embedded foundations by adding the stiffness generated by the footing sides to that generated in the base. In some cases, consideration of the difference in location of the embedment impedance and the basic soil impedance may be included in the analysis. For vertical translation and torsion, the total stiffness and damping results in simple addition of the two values. For horizontal translation and rocking, coupling between the two motions should be considered.

4.2.4 Material damping—Material damping can be incorporated into the stiffness and damping of the footing in several ways. The most direct way is to introduce the complex shear modulus into the governing equations of the soil medium at the beginning of the analysis and to carry out the whole solution with material damping included.

Another way is to carry out the purely elastic solution and then introduce material damping into the results by applying the correspondence principle of viscoelasticity. With steady-state oscillations considered in the derivation of footing stiffness, the application of the correspondence principle consists of replacing the real shear modulus \( G \) by the complex shear modulus \( G^* \).

This replacement should be done consistently wherever \( G \) occurs in the elastic solution. This includes the shear wave velocity (Eq. (3.32)) and the dimensionless frequency (Eq. (4.11)), which consequently become complex. Therefore, all functions that depend on \( a_o \) are complex as well. The substitution of \( G^* \) can be done if analytical expressions for the impedance \( k_i^* \) or constants \( k_i \) and \( c_i \) are available from the elastic solution. With the material damping included, the parameters have the same meaning as before but also depend on the material damping.

The aforementioned procedures for the inclusion of material damping into an elastic solution are accurate but not always convenient. When the elastic solution is obtained using a numerical method, the impedance functions are obtained in a digital or graphical form, and analytical expressions are not available. In such cases, an approximate approach often used to account for material damping multiplies the complex impedance, evaluated without regard to material damping, by the complex factor \((1 + i2\beta_m)\) to determine an adjusted complex impedance

\[
 k_i^* (adj) = (k_i + i\omega_m \cdot c_i) \cdot (1 + 2 \cdot \beta_m \cdot i)
\]

\[
 k_i (adj) = k_i - 2 \cdot \beta_m \cdot c_i \cdot \omega_m \quad \text{and}
\]

\[
 c_i (adj) = c_i + 2 \cdot \beta_m \cdot k_i / \omega_m
\]

where \( \beta_m \) = material damping ratio of the soil, and other terms are as previously defined.

Recognizing the stiffness as the real part of the impedance and the damping as the imaginary term of the impedance, the adjusted stiffness and damping terms considering material damping become

\[
 k_i (adj) = k_i - 2 \cdot \beta_m \cdot c_i \cdot \omega_m \quad \text{and}
\]

\[
 c_i (adj) = c_i + 2 \cdot \beta_m \cdot k_i / \omega_m
\]

where \( k_i \) and \( c_i \) are calculated assuming perfect elasticity, and \( c_i \) includes only geometric damping. Studies indicate that this approximate approach gives sufficient accuracy at low dimensionless frequencies, but the accuracy deteriorates with increasing frequency. Equation (4.22) and (4.23) show that material damping reduces stiffness in addition to increasing damping.

As another approach, variations of Eq. (4.15) are available, which include material damping directly as a distinct parameter (Veletsos and Verbic 1973; Veletsos and Nair 1974).

Finally, some engineers simply add the material damping to the geometric damping otherwise determined by the preceding equations. This approach is more commonly used with the Richart-Whitman formulations and does not alter the stiffness. In such circumstances, broad judgments are often applied at the same time so that if the geometric damping is large, the material damping may be neglected. Similarly, material damping may be included only in those cases where excessive resonance amplification is expected. This simple additive approach is generally recognized as the least accurate of the possible methods.

4.2.5 Pile foundations—Stiffness and damping of piles are affected by interaction of the piles with the surrounding soil. In the past, consideration of this interaction was limited to the determination of the length of the so-called equivalent
cantilever, a freestanding bare pile shorter than the embedded pile. Pile damping was estimated.

The soil-pile interaction under dynamic loading modifies the pile stiffness, making it frequency-dependent. As with shallow foundations, this interaction also generates geometric damping. In groups of closely spaced piles, the character of dynamic stiffness and damping is further complicated by interaction between individual piles known as pile-soil-pile interaction or group effect.

Therefore, recent approaches for determining stiffness and damping of piles consider soil-pile interaction in terms of continuum mechanics and account for propagation of elastic waves. The problem solutions are based on a few approaches, such as the continuum approach, the lumped mass model, the finite element model, and the boundary integral method (Fig. 4.6).

In most cases, the impedance provided to a pile foundation is determined considering only the effects of the piles. Because piles are typically used due to poor surface layer soils, the effects of the soils directly under the cap are often neglected; a settlement gap is assumed to develop. Similarly, the effects of embedment are often neglected. If circumstances indicate that the embedment effect may be significant, the procedures outlined in Section 4.2.3 are often applied.

The basic approach toward pile analysis is to first evaluate the characteristics of a single pile. Once these parameters (stiffness and damping) are established for the single pile, the group effects are determined. Other approaches, such as finite element analysis, may model and consider both effects simultaneously.

4.2.5.1 Single piles—Dynamic behavior of embedded piles depends on frequency and the properties of both the pile and soil. The pile is described by its length, bending and axial stiffnesses, tip/end conditions, mass, and batter (inclination from the vertical). Soil behavior depends on soil properties and the soil’s variation with depth (layering).

Dynamic response of a pile-supported foundation depends on the dynamic stiffness and damping of the piles. These properties for a single pile can be described in terms of either impedance or stiffness and equivalent viscous damping. As previously established, these are related as

\[ k_i^* = k_i + i\omega_m c_i \]  \hspace{1cm} (4-24)

where

\[ k_i^* = \text{complex impedance for the } i\text{-th direction;} \]

\[ k_i = \text{stiffness for the } i\text{-th direction;} \]

\[ \omega_m = \text{circular frequency of motion; and} \]

\[ c_i = \text{damping constant for the } i\text{-th direction.} \]

The engineer can determine the constants experimentally or theoretically. The theoretical approach is commonly used because experiments, though very useful, are difficult to generalize. In the theoretical approach, dynamic stiffness is generated by calculating the forces needed to produce a unit amplitude vibration of the pile head in the prescribed direction (Fig. 4.7).

For a single pile, the impedance at the pile head can be determined from the following

**Vertical translation:**

\[ k_{v1} = \frac{E_p A_p f_{v1}}{r_o} \]  \hspace{1cm} (4-25a)

and

\[ c_{v1} = \frac{E_p A_p}{V_s} f_{v2} \]

**Horizontal translation:**

\[ k_{u1} = \frac{E_p I_p f_{u1}}{r_o} \]  \hspace{1cm} (4-25b)

and

\[ c_{u1} = \frac{E_p I_p}{r_o V_s} f_{u2} \]

**Rotation of the pile head in the vertical plane:**

\[ k_{\psi 1} = \frac{E_p J_p f_{\psi 1}}{R} \]  \hspace{1cm} (4-25c)

and

\[ c_{\psi 1} = \frac{E_p J_p}{r_o V_s} f_{\psi 2} \]

**Coupling between horizontal translation and rotation:**

\[ k_{uw1} = \frac{E_p J_p f_{uw1}}{r_o} \]  \hspace{1cm} (4-25d)

and

\[ c_{uw1} = \frac{E_p J_p}{r_o V_s} f_{uw2} \]

**Torsion:**

\[ k_{\eta 1} = \frac{G_p J_p f_{\eta 1}}{r_o} \]  \hspace{1cm} (4-25e)

and

\[ c_{\eta 1} = \frac{G_p J_p}{V_s} f_{\eta 2} \]

where

\[ k_{ij} = \text{stiffness of pile } j \text{ in the } i\text{-th direction;} \]
\begin{align*}
c_{ij} &= \text{equivalent viscous damping of pile } j \text{ in the } i\text{-th direction;} \\
E_p &= \text{Young’s modulus of the pile;} \\
A_p &= \text{cross-sectional area of the pile;} \\
I_p &= \text{moment of inertia of the pile cross section;} \\
r_o &= \text{pile radius or equivalent radius;} \\
G_p J &= \text{torsional stiffness of the pile;} \\
V_s &= \text{shear wave velocity of the soil; and} \\
f_1, f_2 &= \text{dimensionless stiffness and damping functions for the } i\text{-th direction.}
\end{align*}

Graphical or tabular presentations of the \(f_1\) and \(f_2\) functions are presented in a variety of sources (Novak 1974, 1977; Kuhlmeier 1979a,b) and are included in some software packages. If the pile heads are pinned into the foundation block, then \(k_y = k_{uu} = k_l = 0\) and \(c_y = c_{uv} = c_l = 0\) in the previous formulas, and \(k_u\) should be evaluated for pinned head piles. The vertical constants labeled \(v\) are the same for the fixed and pinned heads. The rotational parameters \((f_{uy1}, f_{uy2} ; f_{uy1} , f_{uy2} ; f_{u1} \text{ and } f_{u2})\) are applicable only if the pile is assumed or designed to be rotationally fixed to the pile cap. In general, the \(f_1\) and \(f_2\) functions depend on the following dimensionless parameters:

- Dimensionless frequency \(a_o = o_o r_o / \sqrt{V_s}\) (note that this value is calculated using the pile radius and is a much smaller value than the \(a_o\) calculated for a complete soil supported foundation);
- Relative stiffness of the soil to the pile, which can be described either by the modulus ratio \(G/E_p\) or the velocity ratio \(V_s/V_c\) in which \(V_c\) is the compression wave velocity of the pile equal to \(\sqrt{E_p / \rho_p}\) with \(\rho_p\) equal to the pile mass density;
- The mass ratio \(\rho / \rho_p\) of the soil and the pile;
- The slenderness ratio \(l_p / r_o\) in which \(l_p\) is pile length;
- Material damping of both soil and pile;
- The pile’s tip restraint condition and rotational fixity of the head; and
- Variation of soil and pile properties with depth.

These factors affecting the functions \(f\) are not of equal importance in all situations. Often, some of them can be neglected, making it possible to present numerical values of function \(f\) in the form of tables and charts for some basic cases.

The pile stiffness diminishes with frequency quickly if the soil is very weak relative to the pile. This happens when the soil shear modulus is very low or when the pile is very stiff. In addition, dynamic stiffness can be considered practically independent of frequency for slender piles in average soil.

The imaginary part of the impedance (pile damping) grows almost linearly with frequency and, therefore, can be represented by constants of equivalent viscous damping \(c_i\), which are also almost frequency independent. Only below the fundamental natural frequencies of the soil layer Eq. (4-17) and (4-18) does geometric damping vanish and material damping remain as the principal source of energy dissipation. Then soil damping can be evaluated using Eq. (4-19) and (4-20). The disappearance of geometric damping may be expected with low frequencies and shallow layers, stiff soils, or both. Apart from these situations, frequency independent viscous damping constants, and functions \(f_2\), which define them, are sufficient for practical applications.

The mass ratio \(\rho / \rho_p\) is another factor whose effect is limited to extreme cases. Only for very heavy piles do the pile stiffness and damping change significantly with the mass ratio.

The Poisson’s ratio effect is very weak for vertical vibration, absent for torsion, and not very strong for the other mode of vibration, unless the Poisson’s ratio approaches 0.5 and frequencies are high. The effect of Poisson’s ratio on parameters \(f_1\) and \(f_2\) can be further reduced if the ratio \(E / E_p\) rather than \(G / E_p\) is used to define the stiffness ratio.

The slenderness ratio, \(l_p / r_o\) and the tip conditions are very important for short piles, particularly for vertical motion because the piles are stiff in that direction. Floating piles (also called friction piles) have lower stiffness but higher damping than end bearing piles. In the horizontal direction, piles tend to be very flexible. Consequently, parameters \(f_1\) and \(f_2\) become practically independent of pile length and the tip condition for ratios \(l_p / r_o\) greater than 25 if the soil medium is homogeneous.

Observations suggest that the most important factors controlling the stiffness and damping functions \(f_1\) and \(f_2\) are the stiffness ratio relating soil stiffness to pile stiffness, the soil profile, and, for the vertical direction, the tip restraint condition.

**4.2.5.2 Pile groups**—Piles are usually used in a group. The behavior of a pile group depends on the distance between individual piles. When the distance between individual piles is large—20 diameters or more—the piles do not affect each other, and the group stiffness and damping are the sums of the contributions from the individual piles. If, however, the piles are closely spaced, they interact with each other. This pile-soil-pile interaction or group effect exerts a considerable influence on the stiffness and damping of the group.

These two basic situations may be treated separately.

**Pile interaction neglected**—When spacing between piles reaches 20 diameters or more, the interaction between piles can be neglected. Then stiffness and damping of the pile group can be determined by the summation of stiffness and damping constants of the individual piles. In many cases, initial calculations are performed neglecting the interaction. An overall group efficiency factor is then determined and applied to the summations.

In the vertical and horizontal directions, the summation is straightforward. For torsion and sliding coupled with rocking, the position of the center of gravity CG and the arrangement of the piles in plan are important. Thus, the group stiffness and damping with respect to rotation derive from the horizontal, vertical, and moment resistance of individual piles and the pile layout.

As the rigid body is rotated, an amount \(\psi\) at the CG (Fig. 4.8), the head of pile \(j\) undergoes horizontal translation \(u_j = \psi v_j\), vertical translation \(v_j = \psi x_j\), and rotation \(\psi_j = \psi\). For the torsional stiffness and damping of the group, the twist \(\eta\) applied at the CG twists the pile by the same angle and translates its head horizontally by the distance equals to \(\eta l_p^2 + \psi^2\) (Fig. 4.9). With these considerations and notations shown on Fig. 4.8 and 4.9, the stiffness and damping constants
of the pile group for individual motions as referenced to the centroid of the pile group are as follows

Vertical translation: \( k_{g_v} = \sum_{j=1}^{N} k_{v_j} \) and \( c_{g_v} = \sum_{j=1}^{N} c_{v_j} \) (4-26a)

Horizontal translation:

\[ k_{g_u} = \sum_{j=1}^{N} k_{u_j} \quad \text{and} \quad c_{g_u} = \sum_{j=1}^{N} c_{u_j} \] (4-26b)

Rotation of the cap in the vertical plane:

\[ k_{g\psi} = \sum_{j=1}^{N} (k_{\psi_j} + k_{\psi_j} x_j^2) \quad \text{and} \quad c_{g\psi} = \sum_{j=1}^{N} (c_{\psi_j} + c_{\psi_j} x_j^2) \] (4-26c)

Coupling between horizontal translation and rotation:

\[ k_{guv} = k_{g\psi u} = \sum_{j=1}^{N} k_{u\psi_j} \quad \text{and} \quad c_{guv} = c_{g\psi u} = \sum_{j=1}^{N} c_{u\psi_j} \] (4-26d)

Torsion about vertical axis:

\[ k_{g\eta} = \sum_{j=1}^{N} [k_{\eta_j} + k_{\eta_j} (x_j^2 + z_j^2)] \quad \text{and} \quad c_{g\eta} = \sum_{j=1}^{N} [c_{\eta_j} + c_{\eta_j} (x_j^2 + z_j^2)] \] (4-26e)

where

\( k_{gij} \) = pile group stiffness in the \( i \)-th direction; and

\( c_{gij} \) = pile group damping in the \( i \)-th direction.

The summations extend over all the piles. The distances \( x_j \) and \( z_j \) refer to the distances from the centroid of the pile group to the individual pile. If the CG is located directly above the pile group centroid, these distances are as indicated in Fig. 4.8 and 4.9. The vertical eccentricity \( y_c \) must be addressed as presented in Section 4.2.6. These stiffness and damping terms, or their impedance equivalents, represent values comparable to the terms developed for a soil-supported foundation in Section 4.2.1.

Pile interaction considered—When piles are closely spaced, they interact with each other because the displacement of one pile contributes to the displacements of others. Studies of these effects call for the consideration of the soil as a continuum.

For static loads, these studies indicate that the main results of the static pile interaction are an increase in settlement of the group, a redistribution of the pile stresses, and, with rigid caps, a redistribution of pile reactions.

The studies of the dynamic pile-soil-pile interaction suggest a number of different observations:

- Dynamic group effects are profound and differ considerably from static group effects;
- Dynamic stiffness and damping of piles groups vary with frequency, and these variations are more dramatic than with single piles; and
- Group stiffness and damping can be either reduced or increased by pile-soil-pile interaction.

These effects can be demonstrated if the group stiffness and damping are described in terms of the group efficiency ratio \( GE \) defined as
where \( k_r \) is the stiffness of individual pile considered in isolation. When the pile-soil-pile interaction effects are absent, \( GE = 1 \). The group efficiency of damping can be defined in the same way.

Dynamic group effects are complex, and there is no simple way of alleviating these complexities. The use of suitable computer programs appears necessary to describe the dynamic group stiffness and damping over a broad frequency range (Novak and Sheta 1982).

The principle alternatives available are replacing the pile group by an equivalent pier, equating dynamic interaction to the static interaction factors, or using the dynamic interaction factors. The equivalent pier may only be applicable for very closely spaced piles and may overestimate damping. The static interaction may be sufficiently accurate for dynamic analysis if the frequencies of interest are low, especially if these frequencies are lower than the natural frequencies of the soil deposit as determined by Eq. (4-17) and (4-18).

### Pile group stiffness using static interaction coefficients

An accurate analysis of static behavior of pile groups should be performed using a suitable computer program (Sharnouby and Novak 1985; Poulos and Randolph 1983). Nevertheless, a simplified approximate analysis suitable for hand calculations can be formulated based on interaction factors \( \alpha \). The interaction factors derive from the deformations of two equally loaded piles and give the fractional increase in deformation of one pile due to the deformation of an equally loaded adjacent pile. The flexibility and stiffness are then established by superposition of the interaction between individual pairs of piles in the group. The approximation comes from neglecting the stiffening effects of the other piles when evaluating the factor \( \alpha \). The accuracy of the approach appears adequate, at least for small to moderately large groups. Poulos and Davis (1980) contains charts for the interaction factors for both axial and lateral loading.

The ratio \( \rho_a = G_{ave}/G_t \) accounts for the variation of soil stiffness with depth, where \( G_{ave} \) is the average shear modulus along the length of the pile, and \( G_t \) is the shear modulus at the pile tip. The stiffness of the pile, relative to the soil, is defined by the stiffness ratio \( \lambda = E_p/G_t \). An approximately linear relationship exists between interaction factors in the vertical direction \( \alpha_v \), and \( \ln(s/d) \) (Poulos 1979). For \( \lambda \geq 500 \), typical of offshore structures, \( \alpha_v \) can be estimated from the following formula (Randolph and Poulos 1982)

\[
\alpha_v = 0.5 \frac{\ln(l_p/s)}{\ln(l_p/d)} \text{ for } s \leq l_p
\]

where

\( s \) = distance between piles;
\( d \) = pile diameter; and
\( \rho_a = G_{ave}/G_t \).

For lateral loading, the pile behavior depends on the length of that part of the pile that deforms appreciably under lateral loading. This critical length may be estimated from the formula (Randolph 1981)

\[
l_c = 2r_c(E_p/G_c)^{27} \tag{4-29}
\]

where

\( l_c \) = critical length of a pile;
\( r_c \) = pile radius;
\( E_p \) = Young’s modulus of the pile; and
\( G_c \) = the average value of dynamic shear modulus of the soil over the critical length.

A few iterations may be needed to find corresponding values of \( G_c \) and \( l_c \).

The interaction factors for horizontal translation \( \psi \) and rotation \( \psi \) may be estimated as (Randolph and Poulos 1982)

\[
\alpha_{vf} = 0.6\rho_c(E_p/G_c)^{17}(r_c/s)(1 + \cos^2\beta) \tag{4-30a}
\]

\[
\alpha_{vH} = 0.4\rho_c(E_p/G_c)^{17}(r_c/s)(1 + \cos^2\beta) \tag{4-30b}
\]

\[
\alpha_{qH} = \alpha_{qH}^3 \text{ and } \alpha_{qM} = \alpha_{qH}^3 \tag{4-30c}
\]

where

\( \rho_c = G_z/G_c \)
\( G_z \) = the shear modulus at depth \( z = l_c/4 \);
\( \alpha_{vf} \) = the horizontal interaction factor for fixed-headed piles (no head rotation);
\( \alpha_{vH} \) = the horizontal interaction factor due to horizontal force (rotation allowed);
\( \alpha_{qH} \) = the rotation due to horizontal force;
\( \alpha_{qM} \) = the rotation due to moment; and
\( \beta \) = the angle between the direction of the loading and the line connecting the pile centers (Fig. 4.10).

When the calculated interaction factor \( \alpha \) exceeds 1/3, its value should be replaced by

\[
\alpha' = 1 - \frac{2}{\sqrt{27a}} \tag{4-31}
\]

This correction is made to avoid \( \alpha \) approaching infinity as \( s \) approaches 0.

For the vertical stiffness of a symmetrical pile group, assuming that all piles carry the same load, the group stiffness may be estimated as

\[
k_{rv} = \frac{\sum k_{rj}}{\sum \alpha_v} \tag{4-32}
\]
in which \( k_{ij} \) is the vertical stiffness of a single pile, and \( \alpha_{ij} \) is the interaction factor between a reference pile \( i \) and any other pile in the group. The summation in the denominator then represents a summation of factors between the reference pile and all other piles in the group. The reference pile should not be in the center or at the periphery and has an interaction factor with itself of \( \alpha_{ii} = 1 \).

If a rigid cap is assumed, which implies the same displacements for all piles, the piles have different individual stiffness values and the following formula is applied (Novak 1979)

\[
\kappa_{g\alpha} = \kappa_{ij} \sum_{i} \sum_{r} e_{ir}
\]

(4-33)

where

\[
e_{ir} = \text{the elements of the inverted matrix } [\alpha_{ij}]^{-1}; \text{ and } [\alpha_{ij}] = \text{matrix of factors between any two piles with diagonal terms } \alpha_{ii} = 1.
\]

The difference between the two previous formulas for \( \kappa_{g\alpha} \) is usually not great. For horizontal stiffness, the approximate correction may be done in a similar fashion using factors \( \alpha_{uf} \) or \( \alpha_{uR} \).

For rotation of a thin rigid cap, the rocking stiffness comes primarily from the vertical stiffness of the piles. This part of the group stiffness becomes

\[
\kappa_{g\psi} = \kappa_{ij} \sum_{r} x_{i} x_{r}
\]

(4-34)

in which \( x \) is the horizontal distance of the pile from the axis of rotation. For thick caps, these corrections can be introduced into the equations of the rotation of the cap in the vertical plane—the case where pile interaction is neglected.

For torsion of the cap, ignoring the contribution from individual pile twisting, the group stiffness can be written analogously as

\[
\kappa_{\eta\eta} = \kappa_{ij} \left[ \sum_{i} \sum_{r} e_{ir} x_{i} z_{r} + \sum_{i} \sum_{r} e_{ir} z_{i} x_{r} \right]
\]

(4-35)

in which \( x \) and \( z \) are the pile coordinates indicated in Fig. 4.9.

If \( \alpha_{i(s)} \) and \( \alpha_{j(z)} \) are horizontal interaction factors between piles \( i \) and \( r \) in direction \( X \) and \( Z \), respectively, \( e_{ir} \) are the elements in \( [\eta_{ij}] = [\alpha_{ij}]^{-1} \).

The static procedure does not offer any guidance as to the effect of interaction on group damping. Group interaction usually increases the damping ratio, not necessarily the damping constant \( c \). To account for this approximately, the group damping constants may be taken as \( c_{g} \approx \sum_{r} c_{r} \). Better estimates may be obtained using dynamic interaction coefficients.

**Evaluation of group effects using dynamic interaction coefficients**—The dynamic interaction factor \( \alpha_{ij} \) is a dimensionless, frequency-dependent, complex number

\[
\alpha_{ij}^* = \alpha_{ij(1)} + i \cdot \alpha_{ij(2)}
\]

(4-36)
determined with respect to the center of gravity. Fig. 4.11 is presented for a simple system where both embedment and bottom support are provided. Evaluation of the forces associated with free-body movements yields an impedance matrix for the system with respect to the center of gravity of

\[
\begin{bmatrix}
K_{uu}^* & K_{uv}^* \\
K_{vu}^* & K_{vv}^*
\end{bmatrix} =
\begin{bmatrix}
k_{uu}^* + k_{uv}^* & - (k_{uu}^* y_e + k_{uv}^* y_e) \\
-(k_{vu}^* y_e + k_{vv}^* y_e) & k_{vv}^* + k_{vu}^* y_e^2 \end{bmatrix}
\]

where

- \(K_{ij}^*\) = impedance in the \(i\)-th direction due to a displacement in the \(j\)-th direction;
- \(y_c\) = distance from the CG to the base support;
- \(y_e\) = distance from the CG to the level of embedment resistance; and
- other terms are as previously defined.

If the CG is not directly over the center of vertical impedance, additional coupling terms between the rotation and the vertical motion are introduced. Thus, most engineers diligently adhere to guidelines to minimize such in-plan eccentricities. In extreme cases, it may be appropriate to develop the \(K^*\) matrix as a full six-by-six matrix due to eccentricities. For other combinations of directions in other coordinate systems, the sign on the off-diagonal terms may change, so proper diligence to sign convention is required. This transformation to the CG may be developed on stiffness and damping terms or based on impedance.

4.3—Vibration analysis

4.3.1 Foundation stiffness—For a concrete block foundation on grade, when the foundation block is thick enough, the foundation block can be considered rigid; a formula used for this judgement is

\[
T > 2 + L_B/30 \quad (SI: T > 0.6 + L_B/30)
\]

where \(T\) is the foundation thickness in ft (m), and \(L_B\) is the greater plan dimension of the foundation block in ft (m). Otherwise, the foundation block is considered flexible in the vibration analysis.

Equation (4-42) is based on engineering judgement and the industry practice for some traditional block-type foundations (Fig. 2.4). The applicability of Eq. (4-42) to large combined block foundations (Fig. 2.5) is not well established and may need to be investigated by the engineer on a case-by-case basis. Finite element dynamic analysis shows that some of these large combined block foundations may not behave as rigid foundations. Analysis of the frequency and dynamic response of such foundations using the finite element methods given in Section 4.3.5 may be appropriate.
In many applications, concrete provides the required inertial mass so that foundations are often massive and rigid. This, in conjunction with the rigid nature of many machines, can greatly simplify dynamic analysis of the machine-foundation system. For equipment on a rigid foundation, when the position of all parts of the system can be described by a single variable at any time, the system can be accurately represented by a SDOF model. For example, pure vertical or torsional motion of a rigid system can be accurately represented by a SDOF model.

Nevertheless, in most rigid machine-foundation systems, the horizontal motion is coupled with the rocking motion due to the system center of gravity (CG) being at one height and the center of resistance (CR) being at a different level. A two DOF model represents the coupled response of the system for this condition. In general, a six DOF (degree of freedom) model is needed to properly represent the dynamic performance of a rigid machine-foundation system. If the foundation is well laid out so that the CG and the CR are positioned over each other (vertically aligned), then the six DOF model mathematically uncouples to become two problems of two DOF (rocking about one horizontal axis coupled with translation along the other horizontal axis for both horizontal directions) and two SDOF problems (vertical motion and torsional motion about the vertical axis).

4.3.2 Single degree-of-freedom system—For a SDOF system, a closed form solution yields the fundamental natural frequency as

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (4-43)$$

where

- $f_n =$ natural frequency in cycles/second;
- $k =$ the dynamic stiffness provided by the supporting media obtained in accordance with Section 4.2, the spring constant of the system; and
- $m =$ mass of the machine-foundation system.

If the particular SDOF system under investigation involved rotational motions, the stiffness term in Eq. (4-43) would be the applicable rocking or torsional stiffness and the mass term would be a mass moment of inertia. Thus, the stiffness and mass terms are specifically associated with a specific direction of motion of the machine-foundation system. The calculated natural frequency is then compared with operating frequencies of the machine to ensure that the frequency criteria as set forth in Section 3.4 are satisfied.

A forced response analysis is performed to determine amplitudes of vibration. The forced response analysis can be a harmonic analysis if the forcing function is harmonic. A closed form solution for a harmonically excited SDOF system yields

$$A = \frac{F_p}{k} \frac{1}{\sqrt{\left[1 - \left(\frac{\omega_p}{\omega_n}\right)^2\right] + \left[2\beta\frac{\omega_p}{\omega_n}\right]}} \quad (4-44)$$

where

- $A =$ displacement amplitude;
- $F_p =$ dynamic force amplitude;
- $k =$ the dynamic stiffness provided by the supporting media;
- $\omega_p =$ circular operating frequency of the machine (rad/s);
- $\omega_n =$ circular natural frequency of the machine-foundation system = $(k/m)^{1/2}$; and
- $\beta =$ system damping ratio $= c/[2(k/m)^{1/2}]$.

A SDOF analysis can be a very effective tool in designing a rigid foundation. When used in conjunction with a parametric study the use of a SDOF in the direction of interest can often bound the solution. Furthermore, the use of an approximate SDOF analysis can show the feasibility of satisfying particular vibration criteria and can provide a rough check on more-detailed analysis.

4.3.3 Two degree-of-freedom system—As mentioned earlier, a SDOF system may not be sufficient to represent a rigid machine-foundation system. For example, a dynamic force from a rotating machine acting on shaft supports causes translation and rocking motions of the system, and a two DOF system is a more accurate representation of the system. For a two DOF system, the following frequency equation can be used in solving for the two natural frequencies

$$\omega_n^4 = \frac{K_{uu} + K_{uv}}{m} \cdot \omega_n^2 + \frac{K_{uu} \cdot K_{vv} - K_{uv}^2}{m \cdot I_q} = 0 \quad (4-45)$$

where

- $\omega_n =$ natural frequencies of the system;
- $I_q =$ mass moment of inertia of the system about the CG;
- $m =$ mass of the machine-foundation system;
- $K_{uu} =$ horizontal spring constant Eq. (4-41);
- $K_{uv} =$ coupling spring constant Eq. (4-41); and
- $K_{vv} =$ rocking spring constant Eq. (4-41).

Amplitudes of motion for the two DOF can be calculated in a manner similar to Eq. (4-44) as a modal combination. When rotational (rocking or torsional) motions of the foundation exist, it is important to compute displacements at the key points of interest machine bearings, for example, by combining the rotational effects with the translational motions. These combined motions may be less than or greater than the motions of the CG, which are determined through the modal combination.

4.3.4 Three or more degree-of-freedom system—Except for cases described in Sections 4.3.2 and 4.3.3, machines supported on flexible or even rigid foundations generally cannot be adequately represented by a single or two DOF model, and a three or more DOF system should be used.

4.3.4.1 Mathematical models—The physical structure can be simplified and represented by a mathematical model for dynamic analysis. The model should include the machine, structure, and stiffnesses of supporting medium (soil or piles). The current state-of-the-art in machine foundation design is limited to linearly elastic theory; nonlinear behavior generally is not considered in routine foundation design.

4.3.4.2 Frequency analysis—A frequency analysis is performed on the mathematical model to calculate natural
frequencies and mode shapes of the system. This calculation is also called normal mode or free vibration analysis. The general dynamic equation of motion is

\[ [M] \ddot{X}(t) + [C] \dot{X}(t) + [K]X(t) = F(t) \]  

(4-46)

where

- \([M]\) = mass matrix;
- \([C]\) = damping matrix;
- \([K]\) = stiffness matrix; and
- \(F(t)\) = time varying force vector.

In Eq. (4-46), \(X\) and its derivatives represent displacement, velocity, and acceleration, respectively, of various points on the machine-foundation system.

For a frequency analysis, the force and damping terms are set to zero. The equation of motion reduces to:

\[ [M] \ddot{X}(t) + [K]X(t) = 0 \]  

(4-47)

Using the normal mode substitutions (common to all classical vibration analysis such as in: Arya, O’Neill, and Pincus 1979; Harris 1996), the following equation is developed

\[ ([k] - \omega_i^2[m]) \Psi_i = 0 \]  

(4-48)

where

- \([k]\) = reduced stiffness matrix;
- \([m]\) = reduced mass matrix;
- \(\omega_i\) = circular natural frequency for the \(i\)-th mode; and
- \(\Psi_i\) = reduced mode shape vector for the \(i\)-th mode.

Solving Eq. (4-48), the natural frequencies (eigenvalues) and mode shapes (eigenvectors) are obtained. An examination is then made to determine how the frequencies associated with the excitation forces generated by machine operation match up with the natural frequencies of the system. If these operating frequencies are close to the computed system natural frequencies, a resonance condition exists. Section 3.4.3 discusses the criteria for frequency separation.

4.3.5 Dynamic analysis using computer codes—Although it is possible to calculate frequencies and responses of a machine-foundation system using equations, practicing engineers find it is often impractical and inefficient to do so, especially for flexible systems with multiple DOF. Many commercially available computer codes, such as ANSYS, DYNA, GTSTRUDL, RISA, SACS, SAP2000, and STAAD, can be used to perform these tasks.

In the computer application, structural beams and columns are generally represented by prismatic elastic members, and concrete floors and mats are represented by plates or solid elements. Stiffnesses of the foundation soil or piles should be properly included in the model to take into account the soil-structure interaction effects. Section 4.2 discusses the impedance of the supporting media. The machine can be quite stiff compared with the structural members. If so, the machine can be represented by a mass point or points concentrated at their centers of gravity and connected to their supports using rigid links. Solid elements with matching densities can also be used to simulate the mass and mass moment of inertia of the machine. If the equipment is not so stiff as compared with the structural members, as is the case for many large gas compressors, its flexibility should be included in the computer model representation.

4.3.4.3 Forced response analysis—In the forced response analysis, Eq. (4-46) is solved for \(X\), the displacements of the machine-foundation system as a function of time. The forced response analysis can take the form of a harmonic frequency analysis or a time history analysis. The selection of method depends on the forcing functions to be evaluated. For the analysis of rotating or reciprocating machinery foundations, the harmonic analysis method is used extensively to determine the steady-state response of a linear structure to a set of given harmonic loads.

Harmonic analysis can be carried out either by direct solution of the equations of motion or by mode superposition techniques. In the direct solution method, the equations of motion can be solved in the time domain. This method does not require a natural frequency analysis (Eq. (4-47) and (4-48)). This numerical method is more general and flexible than the mode superposition method. In the mode superposition method, a natural frequency analysis is performed first. The forced response analysis is then carried out on a mode-by-mode basis, in some cases for a limited number of modes. These modal results are summed to obtain forced response analysis results. The summation may consider the phase relationship between the modes or may conservatively consider only the amplitudes of the modal responses depending on the level of accuracy desired.

The results of the forced response analysis include displacements, velocities, and accelerations. These results are then compared against allowable limits for acceptance. The allowable limits are typically applicable at discrete locations (for example, the equipment bearing, connections to adjacent equipment, and locations where people may be affected). For these multi-DOF models representing structural and machine flexibility, the modeling commonly includes node representations for the locations of interest. The analysis should account for variation in parameters such as structural stiffness, mass, and soil properties.
ACI 318 requirements. Such steps are taken for foundation structures that are very different from conventional building structures (for example, turbine-generator pedestals). For thick sections, special criteria involving location of reinforcing or minimum reinforcing may be identified more in line with mass concrete construction. Such criteria are typically structure specific (for example, only for turbine-generator pedestals or only for foundation slabs over 6 ft (1.8 m) thick) and, thus, are not extendable to the broad class of foundations addressed in this document.

Largely because of the broad range represented in this class of construction, accepted standards have not evolved. For example, there is no specific minimum amount of reinforcement applicable across the board for these designs. In some applications, building code requirements may apply and machine manufacturers may set minimum standards. A minimum concrete strength of 3000 psi (21 MPa) can be applied. For most foundations and foundations supporting API equipment, a strength of 4000 psi (28 MPa) is commonly specified and may be required. In most cases, the quality and durability of the concrete is considered more critical to good performance than strength.

As with most construction, ASTM A 615 Grade 60 reinforcing steel is most commonly used for dynamic machine foundations. Good design practice requires particular attention to the detailing of the reinforcement, including proper development of the bars well into the block of the concrete, avoidance of bar ends in high stress regions, and appropriate cover. Excessive reinforcement can create constructibility and quality problems and should be avoided. Some firms specify a minimum reinforcing of 3.1 lbf/ft³ (50 kg/m³ or 0.64%) for piers (machine support pedestals) and 1.9 lbf/ft³ (30 kg/m³ or 0.38%) for foundation slabs. For compressor blocks, some firms suggest 1% reinforcing by volume and may post-tension the block (Section 4.4.1.5).

Many engineers recommend additional reinforcement, such as hairpin bars, around embedded anchor bolts to ensure long-term performance. The criteria and presentations in ACI 351.2R for static equipment also can be applied to dynamic equipment foundations. 4.4.1.1 Fatigue issues—For many dynamic equipment foundations, the cyclic stresses are small, and engineers choose to not perform any specific fatigue stress calculations. Other equipment can require more significant consideration of cyclic stresses. In such cases, ACI 215 provides guidance, particularly where the flexural characteristics of the foundation are most important.

Some of the methods used by firms to implicitly or explicitly address fatigue include:

- Proportioning sections to resist all conventional loads plus three times the dynamic load;
- Designing such that concrete modulus of rupture is not exceeded while including the inertial loads from the concrete motion. In certain cases, the computed modulus of rupture is reduced by 50% to approximate permissible stresses reduced for fatigue;
- Reducing by as much as 80% the strength reduction factors specified by ACI 318; and

- Recognizing that cracking is less likely in structures built with clean, straight lines and not having re-entrant corners and notches.

4.4.1.2 Dynamic modulus of elasticity—The dynamic modulus of elasticity is stiffer than the static modulus, although not in any simple form. Established relationships suggest that the ratio of dynamic to static modulus can vary from 1.1 to 1.6, with significant variation with age and strength. In practice, engineers treat this strain-rate effect differently. In some firms, engineers perform calculations using the higher dynamic modulus while other firms and engineers consider the difference unimportant and use the static modulus from ACI 318. The distinction is more important for elevated tabletop-type foundations where the frame action of the structure is stiffer if a dynamic modulus is used. The difference can also be important in compressor foundations where the stiffness of the machine frame must be evaluated against the stiffness of the concrete structure. For simple, block-type foundations, the concrete modulus of elasticity has no real effect on the design.

4.4.1.3 Forging hammer foundations—To the committee’s knowledge, there is no current U.S. document addressing design requirements for forging hammer foundations. Most hammer manufacturers are familiar with German DIN Standard 4025. That document is summarized in the following paragraphs for general information.

The required weight of a foundation block sitting on soil should be determined by calculation, and such calculations should consider the effect of vibrations on nearby structures and facilities. One reference equation suggested by DIN 4025 is

$$W_f = 75 \cdot B_r \cdot \left(\frac{v_r}{v_o}\right)^2$$

(4-49)

where

- $W_f$ = weight of the foundation, tons (kN);
- $B_r$ = ram weight, tons (kN);
- $v_r$ = ram impact velocity, ft/s (m/s); and
- $v_o$ = reference velocity 18.4 ft/s (5.6 m/s) from a free fall of 5.25 ft (1.6 m).

This equation assumes an anvil-to-ram weight ratio of 20:1. The foundation weight should be increased or decreased to make up for a lighter or heavier anvil.

The design of the foundation block considers a statically equivalent load determined from the impact energy and characteristics of the forging process in addition to other design basis loads. Minimum reinforcement of the foundation block is set at 1.56 lbf/ft³ (25 kg/m³) or 0.32%. This reinforcement should be distributed in all three directions throughout the block. The upper layer of steel should be capable of resisting 1% of the statically equivalent load applied in any horizontal direction. Bending and shear effects should be addressed in the layout and design of the reinforcement. In large hammer foundations, reinforcement is often installed in all three orthogonal directions and diagonally in the horizontal and vertical planes. Suitable development of the reinforcement is very important.
**4.4.1 Thermal effects**—Some types of dynamic equipment also develop greater than normal thermal conditions, with concrete surface temperatures exceeding 150 °F (66 °C) around and within the foundation. This is especially true for combustion turbines, steam turbines, and compressors. The engineer should address the effects of these thermal conditions in the design phase. Inadequate consideration of the thermal effects can lead to early cracking of the foundation, which is then further increased by the dynamic effects.

Calculation of thermal induced bending requires proper determination of the heat distribution through the thickness of the foundation. ACI 307 provides some guidance that can be extrapolated to hot equipment. ACI 349.1R also provides methodologies that may be transferable to certain machine foundations. Heat transfer calculations can also be performed either one-, two-, or three-dimensionally.

The most effective methods of controlling the thermal effects are:

- Provide sufficient insulation between the hot equipment and the concrete;
- Provide sufficient cover to the reinforcement so that thermally induced cracking neither degrades the bond of the reinforcement nor increases the exposure of the reinforcement to corrosives; and
- Provide sufficient reinforcement to control the growth of thermal induced cracks.

**4.4.1.5 Compressor block post-tensioning**—Some engineers prefer that block-type compressor foundations be post-tensioned to provide residual compressive stress that will prevent the generation of cracks. Shrinkage cracks or surface drying cracks are expected in any concrete block foundation, especially when the water content is excessive.

With the addition of subsequent vibration, these cracks propagate, allowing oil to penetrate the block, and eventually destroy the integrity of the foundation. Post-tensioning puts the block in compression, offsetting the dynamic and shrinkage stresses. When horizontal post-tensioning rods are placed 1/3 the distance from the top of the block, a triangular compression stress distribution can be idealized. This idealization maximizes the compression at the top where it is needed the most. An average pressure of 100 psi (690 kPa) translates to 200 psi (1380 kPa) at the top, providing the necessary residual compression.

Vertical post-tensioning rods are anchored as deep as possible into the foundation mat and are sleeved or taped along their length to allow them to stretch. The embedded end is anchored by a nut with a diameter twice the rod diameter and a thickness 1.5 times the rod diameter. Horizontal rods are nonbonded (sleeved) and anchored at each end of the block through thick bearing plates designed to distribute the load on the concrete. High-strength steel rods are recommended for post-tensioning compressor blocks (Smalley and Pantermuehl 1997).

The concrete should have a 28-day strength of at least 3500 psi (24 MPa), superior tensile strength, and, when cured, be reasonably free of shrinkage cracks. Compliance with ACI 318 Sections 5.8 to 5.13 (Mixing, Conveying, Depositing, Curing, and Hot and Cold Weather Requirements) is commonly mandated for these systems.

**4.4.2 Machine anchorage**—The major components of machine anchorage are the anchor bolts and the support system directly under the machine frame at the anchor bolt location. Support systems range from a full bed of grout to various designs of soleplates and chocks, fixed or adjustable, shown in Fig. 4.12. Additionally, isolation support systems are discussed in Section 4.5. Various styles of anchor bolts are shown in Fig. 4.13.

**4.4.2.1 Performance criteria/anchor bolts**—The structural performance criteria for anchor bolts holding dynamic machinery require that sufficient clamping force be available to maintain the critical alignment of the machine. The clamping force should allow smooth transmission of unbalanced machine forces into the foundation so that the machine and foundation can act as an integrated structure. Generally, higher clamping forces are preferred because high clamping forces result in less vibration being reflected back into the machine. In the presence of unbalanced forces, a machine that has a low clamping force (400 psi [2.8 MPa]) at the machine support points can vibrate more than the same machine with high clamping forces (1000 psi [7 MPa]).

Precision machines in the machine tool industry sometimes have clamping forces as high as 2000 psi (14 MPa) to minimize “tool chatter.” Instead of more refined data, designing for a clamping force that is 150% of the anticipated normal operating bolt force is good practice. A minimum anchor bolt clamping force of 15% of the bolt material yield strength is often used if specific values are not provided by the equipment manufacturer. Higher values are appropriate for more aggressive machines. Clamping force, also known as preload, is developed by pretensioning the anchor bolt.
4.4.2.2 Capacity—The capacity of each anchor bolt should be greater than design loads to provide adequate reserve capacity. Conditions can change over time due to machine wear or changes in operating conditions. Properties as given in the cited ASTM standard specifications of the steels commonly used for anchor bolts are listed in Table 4.3 and 4.4. Because the number and diameter of anchor bolts are determined by the machine manufacturer, the engineer can maximize capacity by specifying the higher-strength steels. The practicable capacity of an anchor bolt is typically 80% of the yield strength, not the full tensile strength.

4.4.2.3 Anchor bolt preload—To avoid slippage under dynamic loads at any interface between the frame and chock and soleplate, or chock and foundation top surface, the normal force at the interface multiplied by the effective coefficient of friction must exceed the maximum horizontal dynamic force applied by the frame at the location of the tie-down.

In general, this requires

\[ F_r = \mu (T_{\text{min}} + W_a) \text{ or } T_{\text{min}} = \frac{F_r}{\mu} - W_a \]  
(4-50)

where

- \( F_r \) = maximum horizontal dynamic force;
- \( \mu \) = coefficient of friction;
- \( T_{\text{min}} \) = minimum required anchor bolt tension; and
- \( W_a \) = equipment weight at anchorage location.

An anchor bolt and concrete anchorage system that has long-term tensile strength in excess of \( T_{\text{min}} \) and maintains preload at or above this tension, coupled with a chock interface whose coefficient of friction equals or exceeds \( \mu \), will withstand the force \( F_r \), to be resisted. A conservative approach neglects \( W_a \) (assumes it to be zero) because distortion of the frame or block may reduce the effective force due to weight at any one anchorage location.

In the case of machines, such as reciprocating gas compressors, gas loads or inertia loads may dictate the required frictional holding capacity \( F_r \) (Section 3.2.3; Smalley and Pantermuehl [1997]), depending on the location of the anchor bolt. Because holes in the frame and cross-head guide establish bolt diameter, the bolt material yield strength determines the maximum possible preload.

The Gas Machinery Research Council (GMRC) research program has set out to develop data for friction between common chock interface materials, including steel/cast iron, steel/steel, epoxy/cast iron, epoxy/steel, epoxy/epoxy, dry, and with oil present, using sizes that come close to those typical of compressor mounting practice (Smalley 1997). This report presents some values for “breakaway” friction coefficients including a range from 0.22 to 0.41 for dry interface between cast iron and various epoxy products and a value of 0.19 for cast iron on cold rolled steel. The presence of oil in the sliding interfaces reduces the friction coefficient for cast iron on epoxy to a range from 0.09 to 0.15 and to a value of 0.14 for cast iron on cold rolled steel. Thus, maintaining an oil free interface greatly enhances frictional holding capacity.

Example:

A separate analysis has shown that each 2 in. (50 mm) diameter anchor bolt of A 193 Grade B7 material for a compressor should carry a maximum horizontal dynamic load of 13,500 lbf (60 kN). What preload tension should be maintained in the anchor bolt?

Using a coefficient of friction of 0.12 and setting the contribution of compressor weight to zero, Eq. (4-50) gives the following minimum tension in the anchor bolt

---

**Table 4.3—Anchor bolt materials—mechanical properties—inch products**

<table>
<thead>
<tr>
<th>ASTM designation</th>
<th>Grade</th>
<th>Diameter, in. (mm)</th>
<th>Tensile strength, min. ksi (MPa)</th>
<th>Yield strength, min., 0.2% offset, ksi (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 36†</td>
<td>—</td>
<td>—</td>
<td>50 to 80 (400 to 550)</td>
<td>36 (250)</td>
</tr>
<tr>
<td>A 307†</td>
<td>A</td>
<td>4 (102) and smaller</td>
<td>60 (414)</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>58 to 80 (400 to 550)</td>
<td>36 (250)</td>
<td>—</td>
</tr>
<tr>
<td>A 193*</td>
<td>B7</td>
<td>2-1/2 (64) and smaller</td>
<td>125 (860)</td>
<td>115 (795)</td>
</tr>
<tr>
<td>F 1554†</td>
<td>36</td>
<td>58 to 80 (400 to 552)</td>
<td>36 (248)</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>55</td>
<td>75 to 95 (517 to 655)</td>
<td>55 (380)</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>105</td>
<td>125 to 150 (862 to 1034)</td>
<td>105 (724)</td>
<td>—</td>
</tr>
</tbody>
</table>

†The values stated in inch-pound units or SI units are to be regarded separately as standard; each system must be used independently of the other. Do not combine values from the two systems.

**Table 4.4—Anchor bolt materials—mechanical properties—metric products**

<table>
<thead>
<tr>
<th>ASTM designation</th>
<th>Grade</th>
<th>Diameter, mm</th>
<th>Tensile strength, min., MPa</th>
<th>Yield strength min., 0.2% offset, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 36M</td>
<td>—</td>
<td>—</td>
<td>400 to 550</td>
<td>250</td>
</tr>
<tr>
<td>A 193M</td>
<td>B7</td>
<td>M64 and smaller</td>
<td>860</td>
<td>720</td>
</tr>
<tr>
<td></td>
<td></td>
<td>M64 to M100</td>
<td>795</td>
<td>655</td>
</tr>
</tbody>
</table>

---
The recommended clamping force (lacking more explicit information) is 150% of this value or 168,750 lbf (750 kN). The nominal area of a 2 in.-diameter bolt is 3.14 in. \(^2\) (20.3 cm\(^2\)). With a yield stress of 105,000 psi (724 MPa), the yield force for the bolt is 330,000 lbf (1467 kN). The required force is 51% of the yield force, which is less than the 80% maximum and greater than the 15% minimum. The bolt should be preloaded to 168,750 lbf (750 kN) and that preload maintained. To compute a minimum required yield stress for this application

\[
T_{\text{min}} = \frac{13,500 \text{ lbf}}{0.12} = 112,500 \text{ lbf} = 500 \text{ kN}.
\]

A material with a yield stress in excess of 67,200 psi (463 MPa) could be substituted for the A 193 Grade B7 material.

**4.4.2.4 Monitoring preload**—Anchor bolts always lose a portion of the preload both in the first 24 h after tightening and then during operation. Usually, at least one retightening is required until the preload stabilizes. Periodic retightening to the original preload level may be required. Bickford (1995) and field observations explain the reasons for this retightening. Some types of machines may require shorter intervals between retightening (after the initial series), but common periods are 6 months to 1 year.

To aid in knowing if the preload loss is excessive, there are electrical and mechanical methods of measuring the preload while the machine is operating. Most electrical methods involve strain gages and read-out devices. Their application is limited to almost laboratory-type conditions. Mechanical methods incorporate the equivalent of a depth micrometer that measures the increase in length of the top of the bolt over a 3 to 4 in. (75 to 100 mm) distance. The UK-patented RotaBolt™ uses the aforementioned principle with a permanent pin and cap in the top of the bolt and achieves an accuracy of 5%. Other products with related mechanisms exist in the marketplace. With any such device, the engineer should seek data and analysis to ensure its fitness for the planned application. There also have been attempts to incorporate both electrical and mechanical methods into “force” washers, but no extensive service history is available at this time. Ultrasonic methods can accurately measure the increase in length of a cleanly terminated bolt (not a J or L) and are available from service providers.

**4.4.2.5 Depth/length/style**—Figure 4.13 shows the various styles of common anchor bolt designs. A good practice is to make the anchor bolt as long as possible so the anchoring forces are distributed lower in the foundation or ideally into the concrete mat below the foundation pier. Anchor bolts that are designed to exactly match a ductile failure criterion, just long enough so that the concrete pullout strength equals the strength of the steel, are too shallow for dynamic machinery foundations. Cracking of the upper concrete has been a common problem when shallow embedment depths are used.

There are additional benefits to using a long anchor bolt. Such systems exhibit greater tolerance to grout creep (that is, less loss of preload from creep). In addition, the lower termination point, in the foundation or in the mat below, moves the potential site for any crack initiation away from the dynamic loads imposed by the machine and away from sources of oil.

In addition to the depth, the engineer should pay attention to the bolt style. J- and L-style bolts can straighten out and pull out of concrete foundations before their maximum tensile capacity is reached. Many engineers prohibit their use with dynamic machinery. Expansion shell anchors should only be used where they have been tested by the manufacturer and approved for the vibratory condition of the particular dynamic machinery application.

**4.4.3 Grout**—The grout chosen should provide the long-term strength to carry the applied load from the machine mounts. For dynamic equipment foundations, engineers often specify cementitious grout (also called hydraulic machine base grout) or polymer grout (epoxy machine base grout is one popular type of polymer grout). For dynamic machines, polymer grouts are often specified because of better resistance to vibration and impact loads and better chemical resistance to process fluids and lubricating oils. The engineer, however, should consider cementitious grouts where they can meet the requirements because they cost less. Cementitious grouts can have compressive strength as high as 6000 psi (42 MPa) but are low in tensile and flexural strength, thereby limiting their use for dynamic machines to smooth running rotating machines such as electric generators.

Polymer grouts can be formulated much stronger in compression than cementitious grouts, with strengths up to 16,000 psi (110 MPa) possible. With machine loads generally less than 1000 psi (7 MPa), high compressive strengths should not be the selection criterion. The higher compressive strength polymer grouts tend to be more brittle and crack readily. Additionally, the engineer should consider other properties such as the modulus of elasticity and creep at operating temperature. Creep, the tendency of materials to exhibit time-dependent deformation, can be a problem, especially with polymer grouts. Their deflection under load increases with time; higher temperatures and thicker grout layers aggravate the tendency. In general, low creep and high modulus of elasticity are desirable. There are products in the marketplace with projected 10-year creep at 140 °F (60 °C) under 600 psi (4.2 MPa) of 0.004 in./in. (mm/mm) or less (tested by ASTM C 1181), and short-term (1 h) modulus of elasticity close to or above 2 × 10^6 psi (13,800 MPa) at 125 °F (52 °C) (ASTM C 580). Creep generally increases as the modulus of elasticity decreases, and both are brand-specific properties. Users are cautioned that increasing grout depth beyond normal thickness of 2 to 4 in. (50 to 100 mm) increases the deformation due to creep and elastic shortening.

Polymer grouts can exhibit slight shrinkage on curing, but such shrinkage is not detrimental to operating life, as
4.5—Use of isolation systems

Isolation systems create a dynamic system with stable, identifiable properties. The consistency of the isolator properties allows other conservative factors to be eliminated. The isolation system provides the stiffness and damping to the system with the equipment and inertia block, if applicable, providing the mass. Most commonly the stiffness provided to the machine-foundation system by an isolation system will be less than that provided by the soil or piles. The isolation system allows the machine-foundation mass to “float” to reduce the transmission of vibrations. The inertia (mass) of the system should resist the dynamic forces from the equipment. The growing practice of base-isolation for the seismic design of buildings has its roots in isolation of machinery vibration.

There are three basic isolator concepts generally used for dynamic machinery applications: rubber pad-type materials, steel springs often combined with viscous dampers, and air mounts. Rubber pad-type materials are generally the stiffest designs and have a nonlinear load deflection relationship. Steel spring systems are softer, providing better isolation. Whereas the springs themselves lack significant damping, these designs can be augmented with viscous dampers, providing damping levels from 5 to 40%, depending on the particular application needs. Air-mount systems are the softest type isolator.

4.5.1 Direct support systems—With direct isolator support of the equipment, the equipment and isolators form the complete dynamic system. The foundation supporting the isolators is subject only to static loads, with the isolators greatly reducing the dynamic force component.

The isolator manufacturer or engineer should arrange the isolators under the equipment in a manner that properly stabilizes the equipment, distributing both stiffness and damping. When isolator manufacturers perform this task, they should provide the foundation engineer with all the necessary design loads for the foundation. If sufficient information is provided (mass distribution, dynamic loads, and
evidenced by over 40 years of use of such products. Most polymer grouts require expansion joints on 3 to 4 ft (900 to 1200 mm) centers because the coefficient of thermal expansion of polymer grouts is greater than concrete and because of the exothermic reaction of the grout during curing. Typical thicknesses for grout are 2 to 4 in. (50 to 100 mm); however, the maximum allowable thickness can be formula specific. Some formulations permit thicknesses up to 18 in. The engineer should assess any mounting system under load for long-term performance considering deflection, creep, applied loads, grout thickness, and operating temperature, relative to the performance criteria in Sections 3.5 and 3.6.

ACI 351.1R provides a more complete explanation of the properties of both polymer and cementitious grouts. Several reports by the GMRC address friction properties, creep properties, and methods of engineering and assessing epoxy grouts and chock material for reciprocating compressor applications (Smalley and Pantermuehl 1997; Smalley 1997; Pantermuehl and Smalley 1997a,b).

4.6—Repairing and upgrading foundations

4.6.1 Introduction—Concrete foundations may need repairing after time because of greater-than-anticipated loads, use beyond the design life, inadequate original design, or inappropriate maintenance. Cracking and deterioration of the concrete that affects the machine’s performance are indicators for repair. Through field experience, cracks larger than 0.016 in. (0.4 mm) have been found to be wide enough to allow penetration by fluids, such as oil, which can thereby cause a crack to grow due to hydraulic action. Furthermore, cracks can cause anchor bolts to loosen and start a vicious cycle (foundation deterioration causes machine deterioration that, in turn, increases foundation deterioration) as illustrated in Fig. 4.14. Because concrete always cracks, it is important to evaluate whether the cracks present a cosmetic problem or a problem in machine performance.

4.6.2 Upgrading—When the engineer considers repairing the foundation, he or she may also consider upgrading the foundation. If the repair is needed because the original design was inadequate, the loads increased, the machine was upgraded, or the original technology is significantly below current standards, then incorporating upgrades into the repair design makes sense. It is illogical to repair a foundation back to its original condition after operation has proven the original design to be inadequate. Application of current technology will likely provide a longer life of trouble-free service than has been experienced with the foundation in the past.

4.6.3 Accomplishing repairs and upgrades—Typically, the condition of the concrete in foundations needing repair is worse in the upper quarter of the foundation where loads are greater and deterioration from process fluids or lubricating oil are more

Fig. 4.14—Foundation deterioration caused by cracking.
likely to be present. Often, a foundation in need of repair can be restored by removal of the top 18 to 24 in. (450 to 600 mm) of old concrete and grout. In this relatively thin upper section, the construction crew can install a strong “bridge” of repair material that can withstand the machine loads and distribute them uniformly to the structure left in place below. Typical elements of an engineered design for upgrading or repairing an old foundation are as follows:

- Vertical post-tensioning to pass through horizontal crack below;
- A heavy reinforcing grid in the top 18 to 24 in. (450 to 600 mm), capable of carrying the machine load and serving as a bridge over the old concrete below;
- Horizontal post-tensioning in one or two directions if there is severe vertical cracking of the remaining concrete;
- Rebuilding of the top foundation with a polymer modified or similar concrete having tensile and flexural strength greater than the concrete it replaces;
- Upgrading of the anchor bolts to increase clamping force, reduce vibration, and reduce the tendency for cracking near the original bolt termination;
- An adjustable machine support system that allows future machine realignment to be easily done. This may also be important if there are uncorrected problems, too costly to fix, in the foundation mat or subsoil below; and
- Increasing the foundation mass, if required, to five times the dead weight of the equipment.

Figure 4.15 shows a cross section of a foundation repair that incorporates several of the above points.

### 4.7—Sample impedance calculations

This chapter presents numerical calculations using the equations from the previous sections in Chapter 4. Only the vertical impedance is included; the other directions follow the same approach. These calculations are not typical of all machine foundations and, as such, broad conclusions about results from alternate equations should not be reached. The results of these various calculations are tabulated in Table 4.5.

#### Given quantities:
- Soil shear modulus $G = 10 \text{k/lin.}^2 = 1,440,000 \text{lbf/ft}^2$;
- Soil Poisson’s ratio $\nu = 0.45$;
- Soil weight density $w = 120 \text{lbf/ft}^3$;
- Material damping $\beta_m = 5\%$, or 0.05;
- Soil is cohesive;
- Mat width $a = 20 \text{ ft}$;
- Mat length $b = 15 \text{ ft}$;
- Machine speed = 350 rpm; and
- Effective embedment depth = 3 ft.

#### Base calculations:

Circular operating frequency $\omega_o$ (definition provided in Section 3.2.2.1b):

$$\omega_o = (350 \text{ rpm})(2\pi \text{ rad/rev})(1/60 \text{ min/s}) = 36.65 \text{ rad/s}$$

Equivalent radius for vertical vibration:

$$R = \frac{ab}{\eta \pi} = \frac{(20 \text{ ft})(15 \text{ ft})}{\pi} = 9.77 \text{ ft} \quad \text{Eq. (4-9a)}$$

Soil mass density $\rho$:

$$\rho = \frac{w}{\text{gravity}} = \frac{(120 \text{ lbf/ft}^3)/(32.2 \text{ ft/s}^2)}{3.73 \text{ lbf-s}^2/\text{ft}^4}$$

Because this machine generates a harmonic dynamic force with a frequency that matches the machine speed, the frequency of the motion will match the operating speed, $\omega_m$ equals $\omega_o$.

Nondimensional frequency:
Method 1: Base vertical impedance—Veletsos: Eq. (4-15)
From Table 4.1:

(Values for 0.45 are interpolated between 0.33 and 0.5.)

\[ \nu = 0.33 \quad \nu = 0.5 \quad \nu = 0.45 \]

\[ \gamma_1 = 0.350 \quad 0.000 \quad 0.103 \]
\[ \gamma_2 = 0.800 \quad 0.000 \quad 0.235 \]
\[ \gamma_3 = 0.000 \quad 0.170 \quad 0.120 \]
\[ \gamma_4 = 0.750 \quad 0.850 \quad 0.821 \]

Parameter \( \chi_v \)

\[ \chi_v = \frac{\gamma_1 (\gamma_2 a_o)^2}{1 + (\gamma_2 a_o)^2} \quad \text{Eq. (4-15c)} \]

\[ \frac{(0.103)(0.235)(0.576)}{1 + [(0.235)(0.576)]^2} = 0.00185 \]

Parameter \( \psi_v \)

\[ \psi_v = \frac{\gamma_1 \gamma_2 (\gamma_4 a_o)^2}{1 + (\gamma_2 a_o)^2} \quad \text{Eq. (4-15c)} \]

\[ \frac{(0.103)(0.235)(0.576)}{1 + [(0.235)(0.576)]^2} = 0.000436 \]

Vertical impedance

\[ k_v^* = \frac{4GR}{(1 - \nu)} [(1 - \chi_v - \gamma_3 a_o^2) + ia_o (\gamma_4 + \psi_v)] \quad \text{Eq. (4-15c)} \]

\[ k_v^* = \frac{4(1,440,000 \text{ lbf/ft}^2)(9.77 \text{ ft})}{(1 - 0.45)} \]

\[ [(1 - 0.00185 - (0.12)(0.576)^2) + i(0.576)(0.821 + 0.000436)] \]

\[ k_v^* = 102,318.545[0.9583 + i0.4731] = 98,100,000 + i48,400,000 \text{ lbf/ft} \]

The stiffness is equivalent to the real part of \( k_v^* \)

\[ k_v = 98,100,000 \text{ lbf/ft or } 8,170 \text{ k/in.} \quad \text{Eq. (4-10)} \]

The damping constant is derived from the imaginary part of \( k_v^* \)

\[ c_v = (48,400,000 \text{ lbf/ft})/(36.65 \text{ rad/s}) \quad \text{Eq. (4-10)} \]

\[ k_v^* = GR(C_{v1} + i a_o C_{v2}) = (1,440,000 \text{ lbf/ft}^2) \quad \text{Eq. (4-16c)} \]

\[ k_v = 105,500,000 + i 55,100,000 \text{ lbf/ft} \]

The stiffness is equivalent to the real part of \( k_v^* \)

\[ k_v = 105,500,000 \text{ lbf/ft or } 8790 \text{ k/in.} \quad \text{Eq. (4-10)} \]

The damping constant is derived from the imaginary part of \( k_v^* \)

\[ c_v = 55,100,000/36.65 = 1,503,000 \text{ lbf-s/ft} \quad \text{Eq. (4-10)} \]

or 125.3 k-s/in.

Calculations using polynomial expansions of \( C_{v1} \) and \( C_{v2} \) support the tabulated constants for this speed of operation.
(\alpha_o = 0.576). For higher speeds, the difference between the constant value and the polynomial are significant.

Including material damping through Eq. (4-23) yields adjusted stiffness and damping terms of:

\[ k_v = 8790 - 2(0.05)(125.3)(36.65) = 8330 \text{ k/in.} \]
\[ c_v = 125.3 + 2(0.05)(8790)/(36.65) = 149.3 \text{ k-s}^2/\text{in.} \]

**Method 3: Base vertical impedance—Richart-Whitman: Eq. (4-12) and (4-13)**

This approach is valid for \( \alpha_o < 1.0 \). Assume the machine weighs 30 kips and the mat is 3.5 ft thick. These assumptions are not significant for the impedance in the vertical direction; the \( k_v \) and \( c_v \) values are not dependent on these weights. For rotational motions there is some minor dependence on the specific assumed values.

The base stiffness is calculated as

\[ k_v = 4GR_o/(1 - \nu) = 4(1,440,000 \text{ lbf/ft}^2) \left(\frac{9.77 \text{ ft}}{1 - 0.45}\right) = 102,300,000 \text{ lbf/ft or 8530 k/in.} \]

The total weight of machine and foundation is

\[ W = (20 \text{ ft})(15 \text{ ft})(3.5 \text{ ft})(0.15 \text{ kcf}) + (30 \text{ kips}) = 187.5 \text{ kips} \]

Either weights \( W, w \) or masses \( m, \rho \) can be used in Eq. (4-12) provided consistency is maintained. The mass ratio for this calculation is

\[ B_v = (1 - \nu)W/[4\omega R_o^3] \quad \text{Eq. (4-12a)} \]
\[ = (1 - 0.45)(187.5 \text{ kips})/[4(0.12 \text{ kcf})(9.77 \text{ ft})^3] \]
\[ = 0.230 \]

Damping ratio (geometric damping)

\[ D_v = \frac{0.425}{\sqrt{B_v}} = \frac{0.425}{\sqrt{0.23}} = 0.886 = 88.6\% \quad \text{Eq. (4-12a)} \]

The total system mass is

\[ M = \frac{W}{g} = \frac{187.5 \text{ kips}}{386.4 \text{ in./s}^2} = 0.485 \text{ k-s}^2/\text{in.} \quad \text{Eq. (4-12a)} \]

The damping constant is calculated as

\[ c_v = 2D_v\sqrt{(k_vM)} = 2(0.886) \quad \text{Eq. (4-14)} \]
\[ \sqrt{(8530 \text{ k/in.})(0.485 \text{ k-s}^2/\text{in.})} = 114.0 \text{ k-s}^2/\text{in.} \]

To include the material damping as a simple addition, the 5% material damping is added to the \( D_v \) value, and Eq. (4-14) is applied. The stiffness is not altered in this approach.

\[ c_v = 2D_v\sqrt{(k_vM)} = 2(0.936) \]
\[ \sqrt{(8530 \text{ k/in.})(0.485 \text{ k-s}^2/\text{in.})} = 120.4 \text{ k-s}^2/\text{in.} \]

**Method 4: Base vertical impedance—Veletsos and Verbic (1973)**

Veletsos and Verbic (1973) presents more complete versions of Eq. (4-15) that include material damping. Short of a complete complex domain solution, this approach is accepted as the best calculation basis. With material damping of 5%, those equations yield:

\[ k_v = 8030 \text{ k/in.} \]
\[ c_v = 133.5 \text{ k-s}^2/\text{in.} \]

**Embedment vertical impedance—Eq. (4-21)**

Because the dimensionless frequency \( \alpha_o = 0.576 \) is in the range of 0.5 to 1.5, the use of Table 4.2 factors is permissible. From Table 4.2, \( S_{v1} = 2.7, S_{v2} = 6.7 \)

Embedment vertical impedance

\[ k_{ev}^* = G_s l \left[ S_{v1} + i\alpha_o S_{v2} \right] \quad \text{Eq. (4-21b)} \]
\[ = (1,440,000 \text{ lbf/ft}^2)(3 \text{ ft})[2.7 + i(0.576)(6.7)] \]
\[ = 11,660,000 + i16,670,000 \text{ lbf/ft} \]

The stiffness is equivalent to the real part of \( k_{ev}^* \)

\[ k_v = 11,660,000 \text{ lbf/ft}, \text{ or 972 k/in.} \quad \text{Eq. (4-10)} \]

The damping constant is derived from the imaginary part of \( k_{ev}^* \)

\[ c_v = (16,670,000 \text{ lbf/ft})(36.65 \text{ rad/s}) \quad \text{Eq. (4-10)} \]
\[ = 455,000 \text{ lbf-s/ft}, \text{ or 37.9 k-s/in.} \]

These values can be adjusted for the material damping effects of the embedment material. The material damping of this side material may be different from the base material. Using Eq. (4-23) yields adjusted stiffness and damping terms of

\[ k_v = (972 \text{ k/in.}) - 2(0.05)(37.9 \text{ k-s/in.})(36.65 \text{ rad/s}) \]
\[ = (972 \text{ k/in.}) - (139.0 \text{ k/in.}) = 833 \text{ k/in.} \]
\[ c_v = (37.9 \text{ k-s/in.}) + 2(0.05)(972 \text{ k/in.})(36.65 \text{ rad/s}) \]
\[ = (37.9 \text{ k-s/in.}) + (2.7 \text{ k-s/in.}) = 40.6 \text{ k-s/in.} \]

The embedment impedance values are directly additive to the base impedance values. For example, combining with the results from the complete solution of Veletsos yields

\[ (k_v)_{\text{total}} = (k_v)_{\text{base}} + (k_v)_{\text{embed}} \]
\[ = (8030 \text{ k/in.}) + (833 \text{ k/in.}) = 8860 \text{ k/in.} \]
\[
(c_v)_{\text{total}} = (c_v)_{\text{base}} + (c_v)_{\text{embed}}
\]
\[
= (133.5 \text{ k-s/in.}) + (40.6 \text{ k-s/in.}) = 174.1 \text{ k-s/in.}
\]

Base impedance values from other calculations could also be used. Most often, consistent approaches are used. The overall results of these various calculations are tabulated in Table 4.5.

### Calculation of displacements

Once the stiffness and damping have been determined, the engineer calculates the maximum response using the approach in Section 4.3. For demonstration, use the Veletsos and Verbic values with the embedment terms added and reduce the damping by 50\% as a rough consideration of possible soil layering effects. Thus, the embedded stiffness is 8860 k/in., and the embedded damping is 87 k-s/in. The total system weight is 187.5 kips. (Refer to Method 3 calculation.)

To calculate motions, determining an excitation force is necessary. Use a given rotor weight of 10 kips and Eq. (3-7) to get

\[
F = 10,000 \text{ lbf} \cdot 350 \text{ rpm}/6000 = 583 \text{ lbf} \quad \text{Eq. (3-7)}
\]

Applying Eq. (4-44) requires calculating \(\omega_n\) and \(\beta\).

\[
\omega_n = (k/m)^{1/2} = (kg/W)^{1/2}
\]

\[
\omega_n = (8860 \text{ k/in.} \cdot 32.2 \text{ ft/s}^2 \cdot 12 \text{ in./ft}/187.5 \text{ k})^{1/2} = 135.1 \text{ rad/s}
\]

\[
\beta = \frac{c}{2\sqrt{km}} = \frac{c}{2\sqrt{kW}}
\]

\[
\beta = \frac{87 \text{ k-s/in.}}{2\sqrt{8860 \text{ k/in.} \cdot 187.5 \text{ k}}/32.2 \text{ ft/s}^2 \cdot 12 \text{ in./ft}} = 0.663
\]

\[
A = \frac{F/k}{\sqrt{(1 - (\omega_n/\omega_p)^2) + (2\beta\omega_n/\omega_p)^2}} \quad \text{Eq. (4-44)}
\]

\[
A = \frac{583/8860}{\sqrt{(1 - (36.65/135.1)^2) + (2(0.663)(36.65)/(135.1))^2}}
\]

\[
= 0.0658 \text{ mils/0.994}
\]

The predicted amplitude motion of this SDOF system is 0.0662 mils or 0.0000662 in. (0.00168 mm). The peak-to-peak motion is 0.1324 mils (3.36 mm) at 350 rpm. For comparison using Fig. 3.10, this motion at this speed is seen to qualify as “extremely smooth.” Similarly on Fig. 3.11, the resultant motion at the operating speed is well below the various corporate and other standards for acceptance.

Although a \(\omega_n\) value is computed for use in Eq. (4-44), this is not actually the system natural frequency because the value has been computed using a stiffness value specific to motion at a specific frequency (350 rpm or 36.65 rad/s). With frequency dependent impedance, as the frequency of excitation increases, the stiffness decreases. Determining the frequency at which maximum response occurs requires an iterative solution across a range of frequencies. For this problem, such an iteration shows that the maximum undamped response (comparable with the aforementioned \(\omega_n\) value) occurs at a speed of 1030 rpm (108 rad/s) and the more realistic, maximum damped response occurs at 1390 rpm (146 rad/s).

### CHAPTER 5—CONSTRUCTION CONSIDERATIONS

#### 5.1—Subsurface preparation and improvement

5.1.1 General considerations—Equipment foundations can be supported on soil, rock, piles, or drilled piers, depending on the geotechnical conditions of the site. The engineer and geotechnical consultant determine the extent of soil investigation and subsurface preparation, which may vary from minimal to extensive. The construction contractor executes the subsurface preparation and the geotechnical engineer verifies it. Adjustments are generally made as required as work progresses.

The engineer should consider the effect of vibratory and impulsive loading on the underlying soils to determine if they are susceptible to dynamic consolidation, particularly under foundations of large dynamic equipment. Thus, additional soil parameters, such as shear wave velocity, dynamic modulus of elasticity, and Poisson’s ratio, may be required from the soil investigation to perform a dynamic analysis (Chapter 3).

5.1.2 Specific subsurface preparation and improvements—The contractor should prepare the site in a way consistent with the assumptions made and parameters used in the foundation analysis. Due to the dynamic nature of loads acting on the foundations, the contractor should pay particular attention to proper compaction and consolidation of the soils.

Specific subsurface preparation and related treatment may be required for one or more of the following reasons:

- If the exploratory borings, field tests and observations, and subsequent laboratory tests dictate the necessity of a subsurface treatment;
- If the exploratory borings reveal nonuniform and heterogeneous conditions with irregularities requiring local remedies; and
- If close inspection of the foundation excavation indicates conditions other than the ones extrapolated from the borings, thereby requiring special preparation and treatment—generally of a localized nature.

Common site-specific subsurface preparations and treatment for these conditions are:

- a) Unstable excavation slopes—Unstable slopes may be stabilized by flattening the slope, benching, dewatering, shoring, freezing, injection with chemical grouts, or supporting with dense slurries;
b) Stratification—Excavations with slopes parallel to the direction of stratification are avoided by flattening the slope or by providing adequate shoring;

c) Wet excavation—During construction, groundwater is normally lowered below the bottom level of the excavation; deep well pumps or well points are commonly used. Another method is to create an impervious barrier around the excavation with cofferdams or caissons, chemical grout injection, sheet piles, or slurry trenches. A sump pit collects groundwater intrusion. The selection of an appropriate method depends on the characteristics of the surface soils encountered, costs, and the preferences of the constructor;

d) Small surface pockets of loose sand—Loose sand pockets are normally compacted to the degree of specified compaction. Alternatively, if the predominant soil is hard, the loose sand may be removed and replaced with flowable fill. Loose sand under dynamically loaded foundations is particularly prone to differential settlement and should be eliminated during construction;

e) Large deposits of loose sands—The loose sands may be stabilized by vibrofloatation or dynamic consolidation, whichever offers an economic advantage. A prediction of long-term settlements considering the vibratory loads may be necessary;

f) Presence of organic material or unconsolidated soft clays—All organic materials and soft clays are normally removed and replaced with suitable, well-compacted fill that provides the characteristics desired for the proper performance of the foundation. Alternatively, piling or drilled piers may be used to carry foundation loads to sound rock;

g) Fissured rock—The extent of fissures is evaluated to determine if remedial treatment is needed. Pressure grouting is a suitable remedy for some types of fissures. In the case of seismic faults, thorough geotechnical and geological evaluation is required to ascertain the potential hazard. Significant hazards exist, relocation of the entire facility to avoid the hazard is a suitable remedy;

h) Irregularly weathered rock—The weathered seams are cleaned and replaced with lean concrete. Alternatively, the foundation may be revised to reach sound rock;

i) Solution cavities in limestone deposits—The voids, if small, are pumped full of grout or, in the case of large holes, lean concrete under a pressure head;

j) Unconsolidated clay—Clays may be preloaded and related settlements monitored. (Early identification is important to gain lead time and avoid slippage in the construction schedule.) Alternatively, piling or drilled piers may be used to carry foundation loads to firm bearing strata;

k) Cold climates—The construction crew should not place foundations on fine-grained soils subject to frost heave. The crew should provide proper drainage by placing a free-draining sand or gravel layer under the foundation to mitigate the possibility of frost heave where such hazard exists. As an alternative, the bottom of the foundation is placed below the frost line.

5.2—Foundation placement tolerances

Foundation placement tolerances depend largely on the type of equipment being supported and are specified by the engineer on the drawings or in the specifications. The construction crew should use templates during concrete placement to support anchor bolts and other embedments that must be precisely positioned.

5.3—Forms and shores

5.3.1 General requirements for forms—Forms and shoring for construction of concrete foundations should follow the recommendations of ACI 347R. As applicable, provisions of ACI 301 should be specified.

5.3.2 Shoring—Shoring should support the concrete loads, impact loads, and temporary construction loads. Transverse and longitudinal bracing may be required to sustain lateral forces.

The formwork engineer should consider wind loads in the shoring design. It is not usually necessary to consider seismic loads due to the limited time shoring is in place. A licensed professional engineer should prepare the design of the formwork and submit it to the design engineer for review.

5.3.3 Shoring systems and formwork for large elevated foundations—For large equipment foundation pedestals, such as turbine-generator foundations, temporary formwork systems are generally used. Less frequently, permanent systems may be used for special applications. The contractor usually selects a temporary support system. The selection is influenced by the erection sequence of the building (if the equipment is enclosed), the equipment installation procedure, and access requirements at the time of placement of the foundation. Some of the permanent systems may affect the design and cost of the foundation. Therefore, the design engineer may wish to consult with building contractors before deciding on a permanent formwork system.

Some of the systems used are:

• Standard construction shoring consisting of temporary shore legs supported by the foundation mat and supporting the soffit forms of a foundation deck;

• Shoring consisting of structural steel beams supported on brackets attached to the foundation columns. The forms rest on top of the beams. Jacking devices are used to lower the beams and forms for removal after the concrete reaches sufficient strength;

• Embedded structural steel shapes (rolled wide flange beams, girders, angles, or channels) supported on the foundation columns and carrying the permanent deck forms. The forms (steel decking) usually rest on the bottom flanges of the steel shapes. Because the steel shapes are embedded in the foundation deck, the design engineer should to be careful to avoid interferences with the reinforcing bars and with other embedments (anchor bolts, plates, pipe sleeves and conduits);

• Embedded structural steel trusses supported on the foundation columns and carrying the permanent deck forms on the bottom chords. The trusses, if specially designed, can also be considered as reinforcing to carry the operating loads acting on the foundation deck. Checking for interferences between the trusses and the
reinforcing bars and other embedments is important to avoid serious construction problems; and

- Precast concrete deck forms supported by the foundation columns. These can be flat bottom “U” and double “U” shapes.

All of these systems, except the Standard Construction Shoring System, allow early access under the foundation deck. The standard shoring system, however, has the least impact on the foundation design. The remaining four systems should be coordinated, in varying degrees, with the foundation design.

In all five cases, the design engineer should review the contractor’s construction procedure.

5.4—Sequence of construction and construction joints

Many large machine foundations are too massive for the concrete to be placed in one continuous operation. Construction joints subdivide large foundations into smaller placement units. Subdivision of large foundations by construction joints also helps reduce internal heat of hydration in concrete and shrinkage cracks in the foundation. To gain maximum benefit, the constructor should place alternate foundation segments and allow them cure and shrink as long as the construction schedule permits before the intervening segments are placed.

The structural integrity of the foundation requires that joints be carefully constructed using accepted practices for construction joints in major concrete structures, such as specified in ACI 301. Project specifications normally require that the constructor obtain the approval of the engineer for construction joint locations and details.

The location of construction joints should follow normal reinforced concrete building practice. Joints in columns should be located at or near the floor line and at the underside for supported beams. If the beams are haunched, the joints should be located at the underside of the deepest haunch. Joints in beams and mats should be located at sections of low stress.

Transverse construction joints should be at right angles to the main reinforcement. Horizontal joints in beams and slabs placed in more than one lift should be supplied with sufficient transverse reinforcing to develop the required horizontal shear capacity by shear friction. Preparation of construction joints should be in accordance with ACI 304R.

Transfer of loads across a construction joint should be provided for by specific means. Tensile loads, for example, should be transferred by extending reinforcing bars across the joint. Transfer of compressive loads can be accomplished by ensuring that the concrete on both sides of the joint is strong and dense. Additional measures are needed to transfer shear loads. Shear keys should be cast in the face of the joint. Alternatively, the face of the joint can be roughened sufficiently for shear loads to be transferred by shear friction. With the latter method, sufficient reinforcing bars should extend across the joint to hold the surfaces of the joint in close contact.

5.5—Equipment installation and setting

5.5.1—Shims, wedges, and bolts represent a typical interface system between the foundation and the machine base. The chosen interface system can be influenced by the machine manufacturer’s recommendations and requirements, the foundation construction procedures, the setting and adjustment of the equipment, and the final tolerances.

Shims, which are usually carbon steel or brass stock in various thicknesses, have both economical and high load-bearing qualities. Shims should be fabricated with rounded corners.

Wedges are usually the double-wedge type and are offered by several mounting-equipment manufacturers. The double wedge mount often has one or more threaded studs for precise vertical adjustment and for locking the sliding wedge into the required position. A lock nut may also be used for locking the main horizontal stud into the final position.

Other types of wedges often used by millwrights include various shaped temporary steel wedges. Temporary wedges are usually tolerance adjustment tools placed before grouting, and they are removed after the hardening of the grout material. Permanent wedge assemblies allow future adjustments on ungrouted equipment bases.

The manufacturer’s drawings should give the required bolt diameters. For construction, the design drawings or specifications should provide bolt diameters, types, overall lengths, threaded lengths, projections, materials, the method of bolt tightening, and required torques. When a specific preload is required by the manufacturer or design engineer, Eq. (5-1) can be used to determine the bolt torque

\[ T_b = W_p K_n d_n \]  

(5-1)

where

- \( T_b \) = bolt torque, lbf-in. (N-m);
- \( d_n \) = nominal bolt diameter, in.;
- \( W_p \) = preload, lbf (Section 4.5.2); and
- \( K_n \) = nut factor for bolt torque (dimensionless).

Typical values of \( K_n \) (often in the range of 0.1 to 0.3) are tabulated in Bickford (1995) and vary with the lubrication and condition of the bolts. Special coatings may require manufacturer’s data.

Required bolt tightening can be accomplished with a post-tensioning jacking procedure, a turn-of-the-nut method, or a calibrated wrench. Post-tensioning jacking is used on the deeper anchorages with nonbonded shanks. When the shank length is embedded in concrete, the turn-of-the-nut method or sequential calibrated wrench tightening is specified. Section 4.4.2.4 presents comments on monitoring the bolt tension. Impact wrenches are not used for tightening a bolt component when part of the anchorage is embedded in concrete because of the extremely high torque and tensile forces delivered by such tools.

5.5.2—Embedments in the concrete include the anchor bolt assemblies previously described, shear lugs, and shear transferring devices. Because shear is a component of a total load transferred to the concrete foundation, steel lugs can be integral parts of the machine base. Such lugs are grouted into shear key grooves previously cast into the concrete base.
5.6—Grouting

5.6.1 Types of grout—There are two basic types of grout: cementitious (cement-based) grouts and polymer (including epoxy-based) grouts. Cementitious grouts are lower in cost, but polymer grouts have higher resistance to chemicals, shock, and vibratory loads.

5.6.2 Applications—ACI 351.1R contains details on the application of grouts. In specifying grout systems, the engineer should consider the different characteristics of each type of grout along with field limitations and match these with specific requirements of the job. In evaluating which cementitious grout should be used, the engineer should consider the placability of the grout and its physical properties: volume change, compressive strength, working time, consistency, and setting time. In evaluating polymer grouts, the engineer should consider the same factors along with creep and the effects of temperature-induced volume changes. The engineer should review the design of the equipment base, accessibility of the grouting location, clearances provided for the grout, and design of anchor bolts. Most of the grouts on the market are premixed, prepackaged materials and contain manufacturer’s instructions on surface preparation, formwork, mixing, placing, and curing procedures.

5.7—Concrete materials (ACI 211.1, ACI 301)

Large machine foundations require special attention to the design and control of the concrete mixture (ACI 207.1R and ACI 207.4R).

Many foundations are massive enough for the heat of hydration of the cement to generate a large thermal differential between the inside and the outside, which may cause unacceptable surface cracking unless steps are taken to reduce the rate of release of this heat. Creep, differential thermal expansion, and shrinkage can cause distortion of the foundation and result in unacceptable changes in machine alignment. It is important to design the concrete mixture to minimize creep and shrinkage and to reduce the thermal expansion of the hardened concrete. Temperatures may be monitored using thermocouples or resistance thermometers. If excessive temperatures are detected, surface cooling systems can be used to provide limited benefits in controlling temperatures (ACI 207.4R). Expansive reaction of the concrete aggregate with alkalies in the cement can be avoided by proper choice of cement and aggregate.

To minimize the rate of release of the heat of hydration, and to control shrinkage and creep, the following steps are normally followed:

- The lowest content of cementitious material consistent with attaining the required strength and durability is used;
- Part of the cement is replaced with fly ash or other pozzolan;
- The placing temperature of fresh concrete is lowered by chilling the aggregate, using chipped ice for mixing water, or both;
- The largest practical size aggregate is used to allow further reduction in the amount of cement;
- Moderate heat cement (Type II) is used;
- A water-reducing agent is used to allow further reduction of the cement factor;
- Low slump and effective vibration are used;
- Concrete placement by pumps, which require concrete mixtures having high amounts of cement and small aggregate sizes, is avoided; and
- Sizes of placements for large foundations are reduced.

High-range water-reducing admixtures may be an appropriate choice because they are consistent with their general applicability to mass concrete applications and heavily reinforced installations for which workability is an issue.

The coefficient of thermal expansion of the hardened concrete can be controlled by the choice because they are of aggregates and because it primarily depends on the coefficient of thermal expansion of the aggregate. When excessive thermal expansion may be a problem, the coefficient of expansion of available aggregates is measured to determine their suitability for the application. (In many regions there may be very limited choices in the types and sources of aggregates.)

Expansion of concrete from alkali-aggregate reaction can be minimized by using a low-alkali cement, by replacing a portion of the cement with a fly ash or nonfly ash pozzolan meeting the requirements of ASTM C 618, and by selecting low-reactivity aggregates. The potential reactivity of aggregates can be evaluated with the procedures and tests described in ASTM C 295, ASTM C 227, ASTM C 289, and ASTM C 586, ASTM C 33 and ACI 225R cover the evaluation methods of the potential reactivity of aggregates.

The cement content should be low enough to help meet heat of hydration requirements but high enough to meet strength, creep, and shrinkage requirements. (It may not be possible to completely solve the heat problem by reducing the heat of hydration. Cooling, small placements, or pozzolans may also be needed.)

5.8—Quality control

Because the foundation for the machine acts as an integral part of the machine-foundation-soil system, an appropriate quality control program should be implemented to ensure that the design requirements are met during construction. ACI 311.4R contains guidance on items to include in the quality program. ACI 311.5R contains guidance on concrete plant inspection and testing of ready-mixed concrete. ACI SP-2 contains general guidance on inspection of concrete. The quality control program should include requirements for control of material quality, the engineer’s approval of critical construction procedures, and onsite verification of compliance with design drawings and project specifications by a qualified field engineer, preferably certified by ACI as a concrete construction inspector.

Design drawings and project specifications should provide foundation requirements to the constructor and field engineer. The field engineer typically is required to report to the design engineer or owner any changes or modifications to the specified design warranted by the conditions in the field. The design engineer or owner should approve any changes in the specified design and document them in accordance with preestablished procedures.
The quality control program and inspections should be thoroughly documented and be available for the owner’s reviews. The quality control program should be consistent with those commonly implemented for construction projects of similar importance. ACI 301 can be cited as part of that quality control program. Laboratories providing testing and inspections should be accredited to the requirements of ASTM E 329.

CHAPTER 6—REFERENCES
6.1—Referenced standards and reports
Documents and standards produced by national and international organizations that are relevant and referenced in this report are listed as follows. In the preparation of this report, currently available editions of these documents were used. Because these documents are subject to frequent revision, the reader is advised to contact the sponsoring agency for the latest versions.

Acoustical Society of America (ASA)
ASA/ANSI S2.19 Mechanical Vibration—Balance Quality Requirements of Rigid Rotors, Part 1: Determination of Permissible Residual Unbalance

American Concrete Institute (ACI)
ACI 117 Standard Specification for Tolerances for Concrete Construction and Materials
ACI 116R Cement and Concrete Terminology
ACI 121R Quality Management System for Concrete Construction
ACI 207.1R Mass Concrete
ACI 207.2R Cracking of Massive Concrete
ACI 207.4R Cooling and Insulating Systems for Mass Concrete
ACI 211.1 Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete
ACI 215R Considerations for Design of Concrete Structures Subjected to Fatigue Loading
ACI 225R Guide to the Selection and Use of Hydraulic Cements
ACI 301 Specifications for Structural Concrete
ACI 304R Guide for Measuring, Mixing, Transporting, and Placing Concrete
ACI 307/307R Design and Construction of Reinforced Concrete Chimneys and Commentary
ACI 311.4R Guide for Concrete Inspection
ACI 311.5R Guide for Concrete Plant Inspection and Field Testing of Ready-Mixed Concrete
ACI 318/318R Building Code Requirements for Structural Concrete and Commentary
ACI 347R Guide to Formwork for Concrete
ACI 349.1R Reinforced Concrete Design for Thermal Effects on Nuclear Power Plant Structures
ACI 351.1R Grouting Between Foundations and Bases for Support of Equipment and Machinery
ACI 351.2R Foundations for Static Equipment
ACI SP-2 ACI Manual of Concrete Inspection

American Petroleum Institute (API)
API 541 Form-Wound Squirrel Cage Induction Motors—250 Horsepower & Larger
API 610 Centrifugal Pumps for Petroleum, Heavy Duty Chemical, & Gas Industry Services
API 612 Special-Purpose Steam Turbines for Petroleum, Chemical, & Gas Industry Services
API 613 Special-Purpose Gear Units for Petroleum, Chemical, & Gas Industry Services
API 617 Centrifugal Compressors for Petroleum, Chemical, and Gas Industry Services
API 618 Reciprocating Compressors for Petroleum, Chemical, and Gas Industry Services
API 619 Rotary-Type Positive Displacement Compressors for General Refinery Services
API 684 Tutorial on the API Standard Paragraphs Covering Rotor Dynamics and Balance (An Introduction to Lateral Critical and Train Torsional Analysis and Rotor Balancing)
API 686 Recommended Practice for Machinery Installation and Installation Design

American Society of Civil Engineers (ASCE)
ASCE 7 Minimum Design Loads for Buildings and Other Structures

ASTM International
ASTM A 36/ A 36M Standard Specification for Carbon Structural Steel
ASTM A 193 Standard Specification for Alloy-Steel and Stainless Steel Bolting Materials for High-Temperature Service
ASTM A 307 Standard Specification for Carbon Steel Bolts and Studs, 60,000 PSI Tensile Strength
ASTM A 615 Standard Specification for Deformed and Plain Billet-Steel Bars for Concrete Reinforcement
ASTM C 33 Standard Specification for Concrete Aggregates
ASTM C 295 Standard Guide for Petrographic Examination of Aggregates for Concrete
ASTM C 580 Standard Test Method for Flexural Strength and Modulus of Elasticity of Chemical-Resistant Mortars, Grouts, Monolithic Surfacings, and Polymer Concretes
ASTM C 586 Standard Test Method for Potential Alkali Reactivity of Carbonate Rocks for Concrete Aggregates (Rock Cylinder Method)
ASTM C 618 Standard Specification for Coal Fly Ash and Raw or Calcined Natural Pozzolan for Use as a Mineral Admixture in Concrete
ASTM C 1181 Standard Test Methods for Compressive Creep of Chemical-Resistant Polymer Machinery Grouts

ASTM D 4015 Standard Test Methods for Modulus and Damping of Soils by the Resonant Column Method

ASTM E 329 Standard Specification for Agencies Engaged in the Testing and/or Inspection of Materials Used in Construction

ASTM F 1554 Standard Practice for Anchor Bolts, Steel, 36, 55, and 105-ksi Yield Strength


Deutsches Institut für Normung (DIN)
DIN 4024 Part 1 Machine Foundations: Elastic Supporting Constructions for Machines with Rotating Masses
DIN 4024 Part 2 Machine Foundations: Rigid Supporting Constructions for Machines with Periodic Excitation
DIN 4025 Foundations for Drop Forging Machinery
DIN 4150 Part 1 Vibrations in Buildings: Prediction of Vibration Parameters
DIN 4150 Part 3 Vibrations in Buildings: Effects on Structures

Federal Emergency Management Administration (FEMA)

International Conference of Building Officials (ICBO)
IBC International Building Code
UBC Uniform Building Code

International Standards Organization (ISO)
ISO 1940-1 Mechanical Vibration—Balance Quality Requirements of Rigid Rotors—Part 1: Determination of Permissible Residual Unbalance
ISO 10816-2 Mechanical Vibration—Evaluation of Machine Vibration by Measurements on Non-Rotating Parts—Part 2: Large Land-Based Steam Turbine Generator Sets in Excess of 50 MW
ISO 10816-3 Mechanical Vibration—Evaluation of Machine Vibration by Measurements on Non-Rotating Parts—Part 3: Industrial Machines with Nominal Power Above 15 kW and Nominal Speeds Between 120 r/min and 15,000 r/min when Measured In Situ
ISO 10816-6 Mechanical Vibration—Evaluation of Machine Vibration by Measurements on Non-Rotating Parts—Part 6: Reciprocating Machines with Power Ratings above 100 kW

Verein Deutscher Inginieure (VDI)
VDI 2057 Effect of Mechanical Vibrations on Human Beings

These publications may be obtained from these organizations:

Acoustical Society of America
335 East 45th St.
New York, NY 10017-3483
web: http://asa.aip.org

American Concrete Institute
Order Desk
1220 L St. NW
Washington, D.C. 20005-4070
web: http://www.concrete.org

American Petroleum Institute
1220 L St. NW
Washington, D.C. 20005-4070
web: http://www.api.org

American Society of Civil Engineers
1801 Alexander Bell Dr.
Reston, VA 20191
web: http://www.asce.org

ASTM International
100 Barr Harbor Dr.
West Conshohocken, PA 19428
web: http://www.astm.org

Deutsches Institut für Normung (DIN)
Burggrafenstrasse 6
DE-10772 Berlin
Germany
web: http://www.din.de

Federal Emergency Management Agency
Building Seismic Safety Council
1090 Vermont Avenue, Suite 700
Washington, D.C. 20005
web: http://www.bssconline.org
Foundations for Dynamic Equipment

6.2—Cited references


### 6.3—Software sources and other references

**ANSYS**—ANSYS, Inc., Global Headquarters
Southpointe
275 Technology Drive
Canonsburg, PA 15317
Web: http://www.ansys.com

**DYNA**—Dept. of Civil Engineering
Geotechnical Research Centre
London, Ontario, N6A 5B9
Canada
Web: http://www.engga.uwo.ca/civil/grc/computer.html

**GTSTRUDL**—Georgia Tech—CASE Center
School of Civil & Environmental Engineering
Atlanta, GA 30332-0355
Web: http://www.gtstrudl.gatech.edu/

**RISA**—RISA Technologies
26632 Towne Centre Drive, Suite 210
Foothill Ranch, CA 92610
Web: http://www.risatech.com/

**SACS**—Engineering Dynamics, Inc.
2113 38th Street
Kenner, LA 70065
Web: http://www.sacs-edi.com
1995 University Ave., Suite 540
Berkeley, CA 94704
Web: http://www.csiberkeley.com

STAAD—Research Engineers International, Headquarters
22700 Savi Ranch Parkway
Yorba Linda, CA 92887-4608
Web: http://www.reiworld.com

Matlock, H.; Foo, H. C.; and Bryant, L. M., 1978, “Simulation of Lateral Pile Behavior Under Earthquake Motion,”
Earthquake Engineering and Soil Dynamics: Proceedings of the ASCE Geotechnical Engineering Division Specialty Conference, June 19-21, 1978, Pasadena, CA, ASCE.


6.4—Terminology
The following terms are common terminology for dynamic equipment foundations. These terms may differ from standard ACI terminology (ACI 116R).

acceleration—Time rate of change of velocity; a vector quantity measured in units of gravity, in./s/s (m/s/s).

amplitude—Maximum value of an oscillating quantity measured from the position or level of equilibrium (zero-to-peak value). Amplitude may be expressed in terms of displacement, velocity, acceleration, force, or any other time-varying quantity. In general, the terms “single,” “zero-to-peak,” and “peak” are unnecessary modifiers for “amplitude.” However, the vibration measurement industry often reports displacement measurements in terms of “peak-to-peak” or “double” amplitude, although these are mathematical misnomers. Thus, the “single” amplitude modifiers are used to provide specificity.

analysis, dynamic—A general term referring to the process of analyzing a vibratory system for evaluating its natural frequencies, mode shapes, and responses to excitation.

analysis, forced response—That part of dynamic analysis carried out to evaluate the response of a vibratory system subjected to general excitation. The response of the system is described by the complete integral of the governing differential equation. Harmonic and time-history analyses are subsets of the more general forced response analysis.

analysis, harmonic—The dynamic analysis of a vibratory system subjected to sinusoidal-type excitation. Harmonic analysis neglects the initial conditions and involves only the particular solution to the governing differential equation of motion.

analysis, modal—The dynamic analysis of a multidegree-of-freedom system by which the responses in each mode of vibration are determined separately and then superimposed to obtain the total response.

analysis, steady-state response—A term synonymous with “harmonic analysis.”

analysis, time history—The dynamic analysis in which the response of a vibratory system is evaluated based on a set of specified time-varying excitation parameters such as force, acceleration, or displacement. The excitation input is in the form of a time history for a specified duration of interest. Time history analysis is also known as transient response analysis.

center of gravity—That point within a body through which, for any orientation of the body, passes the resultant of the gravitational forces (weights); in common practice, equivalent to center of mass or mass centroid; common notation: CG.

chock—Part of the interface between the machine and the concrete foundation that provides final alignment capability and adjustability.

critical speed—The speed (usually in rpm) of a rotating element at which the element exhibits dynamic instability and large amplitude motion. This motion develops when the angular speed of the rotating element matches one of the rotating element’s natural frequencies.

damping constant—The ratio between the damping force and the system velocity. For viscously damped systems, the ratio is constant (independent of frequency). Measured in units of lbf-s/in. or (N-s/m); common notation: c.

damping ratio—The ratio of the actual system damping to the system’s critical damping. A system’s critical damping is the minimum amount of viscous damping that leads to a system not oscillating in free vibration situations.

damping, geometric—The dissipation of energy resulting from a reduction in intensity of mechanical waves radiating from a vibration source and propagating through an elastic medium, also called radiation damping.

damping, hysteretic—See material damping.

damping, material—The dissipation of energy within material as a result of internal friction. The damping force is directly proportional to the displacement, independent of frequency, and in phase with the velocity of the system. Material damping may also be called hysteretic damping, structural damping, or internal damping. For consistency, use of the term “material damping” is recommended.

damping, radiation—A variant term for geometric damping.

damping, viscous—A type of damping in which the damping force is directly proportional to the velocity of the system at all times. Thus, the damping force is in phase with the velocity.

degrees of freedom—The independent coordinates used to specify the position of a body (or system) in motion at any time. (See also single degree-of-freedom system and multi-degree-of-freedom system.)

displacement—A vector quantity that specifies the position of a body in motion with respect to a defined system of reference, usually a position of rest or equilibrium. Rotational (angular) displacement is measured in radians. Translational displacement is measured in inches or meters. Peak-to-peak displacement reflects the total travel that a point undergoes through a full cycle of vibration. For simple harmonic motion, this is twice the displacement amplitude. This quantity is commonly reported by vibration measuring equipment and is often incorporated into acceptance criteria.
dynamic force—See excitation force; for discussion of forces, also see the definition of excitation.

elastic half-space—A term used to describe an idealized semi-infinite medium such as soil mass.

excitation—Mechanical disturbance imparted to a physical system by the direct application of an external force or support motion.

excitation force—generated by the source of excitation. These are frequency-dependent forces produced by dynamic equipment. Force is usually express as pounds-force or Newtons.

In English Units (Gravitational System): pound-force (lbf) is the unit of force that, when acting on a mass of one kilogram, will impart to it an acceleration of 1.0 ft/s/s. One kilo-Newton (KN) is 1000 lbf.

In SI Units (absolute System): newton (N) is the unit of force that, when acting on a mass of one kilogram, will impart to it an acceleration of 1.0 m/s/s. One kilo-Newton (KN) is 1000 N.

In North America, pound is often taken to mean pound-force (lbf). It may, however, sometimes cause confusion with pound-mass (lbm).

frequency—Number of complete vibrations per second. The unit is cycles per second (cps) or Hertz (Hz). The latter term is more commonly used; common notion: f.

frequency, circular—Frequency expressed in radians per second (rad/s). Sometimes called angular frequency or vibrational frequency; common notation: ω.

frequency, excitation—The frequency of the vibration source.

frequency, fundamental—The lowest natural frequency of the vibratory system. This term may be used in a direction-dependent manner so that the vertical fundamental frequency may differ from a lateral fundamental frequency.

frequency, natural—The frequency at which a vibratory system will vibrate without the influence of any external force or damping. May be expressed in terms of frequency (cps or Hz) or circular frequency (rad/s).

frequency ratio—The ratio of excitation frequency to the natural frequency of the system.

frequency, resonant—The frequency at which the dynamic magnification is a maximum.

geophone—A velocity transducer often used in soil tests.

inertia block—A solid, heavy piece, often made of concrete and assumed to be rigid, that is used as part of the foundation system principally for its weight contribution.

machine foundation—This term has always caused some confusion as to exactly what it means. To the structural or geotechnical engineer, foundation is the concrete substructure resting on or below ground. The machine manufacturer, however, has traditionally used this term to designate the structural supporting system below the soleplate of the machine. Thus, the elevated concrete structure supporting a machine such as the paper machine is still called a machine foundation by the manufacturer; although, such foundation can be as high as 25 ft above ground. Such confusion may lead to misinterpretation by the structural engineer of the manufacturer’s design parameters and criteria. Within this document the manufacturer’s understanding is used: the supporting structure below the soleplate of the machine.

mass—The physical quality of a body reflecting the quantity of matter inherent in the body. A specific body’s mass does not vary whereas the body’s weight will change depending on the local gravitational field (on earth versus on the moon). In vibration applications, a body’s mass is a critical parameter. Units associated with mass are typically grams or kilograms in absolute SI units and pound-mass (lbm) or slugs in FPS systems. A slug is a mass that, when subjected to a force of one pound, accelerates one ft/s^2. (1 slug = 1 lbf-s^2/ft). A pound-mass, when subjected to a force of one pound, accelerates 32.2 ft/s^2 (9.81 m/s^2). That is, on earth, a mass of one lbm weighs one lbf. Further explanation can be found in ASTM SI-10.

mathematical model—The idealized representation of a physical system for mathematical treatment and computer analysis. The level of complexity should be compatible with the required degree of accuracy. For dynamic analysis, a mathematical model may include the following information:

• Geometry of the structure (joint coordinates);

• Types of members (such as beam, truss, plate, membranes, and solids), and physical properties (such as section, sizes, and thickness);

• Concentrated masses;

• Member connectivity, end-fixity, and boundary conditions;

• Material properties (such as mass densities and elastic constants);

• Damping; and

• Excitation forces, their locations, magnitudes, and frequencies.

mode of vibration—A characteristic deflection shape of a structure corresponding to a specific natural frequency of the system, also known as an eigenvector.

modulus of elasticity—The ratio of axial stress to axial strain, also known as Young’s Modulus.

modulus of subgrade reaction—The soil pressure per unit displacement of a foundation (usually in the vertical direction), also called coefficient of subgrade reaction. This soil property is strain-rate dependent, therefore, different values apply for static and dynamic loading.

modulus, shear—The ratio of shear stress to shear strain, also known as modulus of rigidity; however, the term shear modulus is more common and is always used in discussion involving soils.

multi-degree-of-freedom system (MDOF)—A vibratory system that requires two or more independent coordinates to completely specify its motion. (See also single degree-of-freedom system.)

resonance—The state of steady-state vibration in which the excitation frequency is equal to or close to a system damped natural frequency. In this state, system displacements are usually amplified (for lightly damped systems) and can be very sensitive to changes in the excitation frequency, system stiffness, or system mass.

response—The motion of a vibratory system resulting from excitation. The motion can be mathematically described in terms of displacement, velocity, acceleration, or other parameters.
root-mean-square (rms) value—A time-weighted average measurement of a particular quantity (force, displacement, velocity, or acceleration). Rms velocity is a useful measure of vibration severity in cases where the vibration is complex such that displacement, velocity, and acceleration are not clearly related. For general functions, a rms value is calculated as

$$v_{rms} = \sqrt{\frac{1}{T} \int_{0}^{T} v^2(t) dt}$$

For single-frequency, harmonic functions, an rms value is equal to 0.707 of the function amplitude.

single degree-of-freedom system (SDOF)—A vibratory system the motion of which can be completely specified by a single coordinate. (See also Multi-degree-of-freedom system.)

soleplate—The member that interfaces to or between the machine and the supporting structure.

stiffness—The ratio of the applied force to the resulting foundation movement, expressed in lb/in. or (N/m); common notation: \(k\).

transmissibility—A measure of the ability of a dynamic system to transmit energy from a source to a location, commonly computed as a transmissibility ratio.

unbalanced force—See excitation force.

velocity—A vector quantity that identifies the time rate of change of motion. Translational velocity is measured in in./s (m/s), rotational velocity is rarely used.